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THE STRESS CONCENTRATORS AT THE BIMATERIAL INTERFACES

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Ústav mechaniky těles, mechatroniky a biomechaniky

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The stress concentrators at the bimaterial interfaces

Koncentrátory napětí v blízkosti bimateriálových rozhraní

Habilitační práce

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Contents

Tomáš Profant se narodil 15. června 1975 ve Vsetíně. V letech 1993–1997 studoval na Fakultě strojního inženýrství VUT v Brně obor Matematické inženýrství. Na stejné fakultě v roce 1998 zahájil prezenční formu doktorského studia na Ústavu mechaniky těles, mechatroniky a biomechaniky, které však přerušil od dubna roku 2000 do května roku 2002 a pokračoval v jeho distanční formě. V této době pracoval jako technik úseku normování práce (ufficio tempi e motodi) v Nová Mosilana a.s., člena italského koncernu Marzotto. Po návratu na fakultu v listopadu roku 2003 obhájil dizertační práci *Interakce mikrotrhlin s částicemi druhé fáze.*

Svou odbornou a pedagogickou činnost zahájil od roku 2003 jako odborný asistent na Ústavu mechaniky těles, mechatroniky a biomechaniky Fakulty strojního inženýrství VUT v Brně. Jeho vědecko-výzkumná činnost se zaměřuje na lomovou mechaniku heterogenních materiálů a kompozitů. Spolupracuje s Ústavem fyziky materiálů AV ČR. Byl a je řešitelem nebo spoluřešitelem projektů GAČR 101/05/P290, 101/08/0994 a 108/10/2049 a účastní se také na řešení dalšího projektu Grantové agentury ČR. Výsledky jeho vědecké práce jsou publikovány v 41 článcích v odborných časopisech nebo sbornících konferencí a seminářů.

Jeho pedagogická činnost je spjata s Fakultou strojního inženýrství VUT, kde učí jako cvičící příp. přednášející předmětu Pružnost a pevnost I, příp. Pružnost a pevnost II a Statika. Svou pedagogickou činnost také obohatil několika pobyty na zahraničních univerzitách v rámci programu Socrates-Erasmus v Itálii a Švédsku.

Kromě vědecké činnosti se ve volném čase věnuje dětské ilustraci a reklamní grafice. Je autorem ilustrací k několika pedagogicky zaměřeným publikacím vydaných v nakladatelstvích Didaktis a Computer Press. Podílel se také např. jako ilustrátor na výstavě Nanotechnologie konané ve spolupráci FSI VUT v Brně a brněnským Technickým muzeem.

An introduction

Basic mathematical tools of the fracture mechanics is based on the Kolosov's doctoral thesis at University of Dorpad (present Tartu), Estonia, [3]. The nonlinear phenomena was taken into account by Griffith [4] by using energy considerations and the concept of surface energy. Orowan [5] extended the Griffith's theory by inclusion of all dissipative energy, essentially the surface energy and plastic work, because the Griffith's theory avoids an analysis of the crack edge neighborhood. Irwin [6] introduced the concepts such as the *stress intensity factor* and the *energy release rate*. The *critical stress intensity factor* (*fracture toughness*) became concept that laid the foundation of the *linear* (*elastic*) *fracture mechanics* (LEFM). Barenblatt [7, 8] introduced the concept of *autonomy* of the field near the crack edge. Finally, among other significant contributions, it must be mentioned the Rice's one [9], which introduced the J-integral concept, a path-independent integral, for crack analysis. The J-integral concept laid the foundation for the nonlinear fracture mechanics.

A very powerful tool for crack modeling is the theory of plane elasticity, especially, the theories based on the theory of the complex function like Muskhelishvili's complex potentials of isotropic elasticity, see Muskhelishvili [18], and Lekhnitskii or sextic Stroh formalism dealing with the anisotropy, see Ting [19]. Anisotropic elastic material considered in two dimensional deformations may have as many as fifteen elastic constants. In contrast to this, the Lekhnitskii-Stroh formalism replaces the fifteen elastic constants by three eigenvalues and corresponding eigenvectors. The Lekhnitskii-Stroh formalism is mathematically elegant and technically powerful. The Lekhnitskii formalism was laid down in 1950, [20, 21], and the newer Stroh formalism were laid down in 1958, [22, 23].

The microcracking is the particular fracture problem, where the cracks are relatively short and are growing in the neighborhood of some stress rising features causing a relatively steep stress gradients. In this case, it is practicable to represent the geometry of the problem in an idealized way and all the previously discussed formalisms allow one to find the Green's functions of these problems, which can be subsequently utilized by the distributed dislocation technique to model the evolution of the developing micro-cracks. The distributed dislocation technique is based on the pioneering works of Eshelby [26, 27], Erdogan [49, 50] and many others, and offers a precise solutions contrasted with the approximative solutions received by the finite element method in which the geometry is modeled exactly.

On the other hand, to exploit the finite element method with the elimination of its inappropriate behavior near the stress concentrators, it can be applied the two-state integral procedure, especially the so-called Ψ-integral. The Ψ-integral follows from the Betti's reciprocity theorem in the absence of the body forces and enables to determine the local stress field parameters in the vicinity of the crack or notch tip using the deformation and the stress field in the remote points, where the numerical results obtained e.g. using finite element analysis are more accurate.

In the presented work, there are summarized the results of the articles published during the years 2003–2010 under prof. Michal Kotoul leadership or in conjunction with Jan Klusák from Institute of physics of materials, Academy of sciences of the Czech republic. The results discussed in the following chapters are received from the application of the methods mentioned above.

The author is grateful to prof. Michal Kotoul for his leadership and the possibility to participate on his projects and scientific research. The author is also grateful to his colleagues Oldřich Ševeček, Ph.D. and Jan Klusák, Ph.D. for the brave collaboration. The thanks must be also expressed to the author's close colleagues from the Institute of solid mechanics, mechatronics and biomechanics.

Above all, author has to expresses many thanks to his wife, Saša, and his son, Bartoloměj.

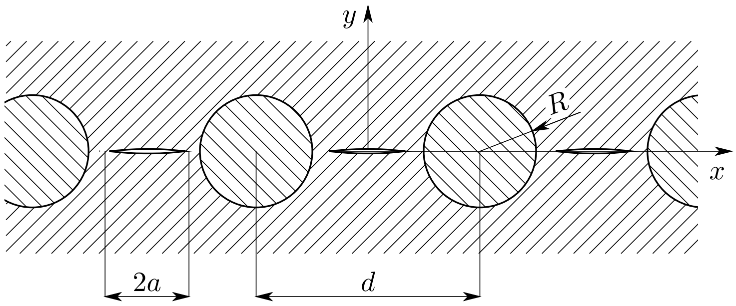

Figure 1: The periodic array of the inclusions and microcracks. The tips of the microcracks lie in the matrix.

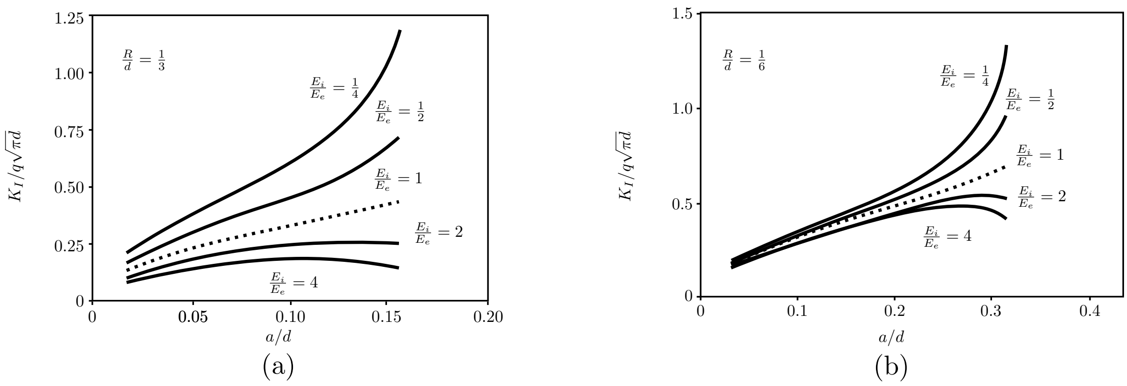

Figure 2: The normalized stress intensity factor $K_I/(q\sqrt{\pi d})$ due to the remote loading q as a function of the normalized microcrack length a/d for various ratios of the elastic moduli of the inclusions E_i and the matrix E_e and for the ratio (a) $R/d = 1/3$, (b) $R/d = 1/6$.

1 The cracks and notches at the bimaterial interface

The important area of the fracture mechanics application is covered by the study of the ceramics and composites behavior. As an example of the stress intensity factor application, the tensile strength of the ceramics can be estimated using the collinear microcrack model, see [a2, a4, a5, a8, a10, a11]. This model allows one to assess the behavior of the formation of the microcracks in loaded ceramic matrix containing the inclusions, whose elastic properties differ from the elastic properties of the matrix. Because of the analytical description of the problem, it is considered the simplest case of the microcrakcs and inclusions configuration - the periodic array of collinear microcracks and inclusions placed between the neighboring microcrack tips. As the first, it is supposed that the microcrack tips lie in the matrix, see Fig. 1. The value of the stress exponent singularity for this microcracks and inclusions configuration is 1/2 and it is reasonable to study the stress intensity factor K_I defined in homogeneous solid, which is given by the following formula

$$K_I = \frac{2\mu_e}{(1+\kappa_e)}\left[d\tan\left(\frac{\pi a}{d}\right)\right]^{1/2} g(1), \tag{1}$$

where μ_e is the matrix shear modulus, $\kappa_e = 3 - 4\nu_e$ and ν_e is the matrix Poisson's ratio. Parameters a and d are the half-length of the crack and the distance between the centers of neighboring inclusions. The function $g(s)$ is the linear combination of Jacobi polynomials

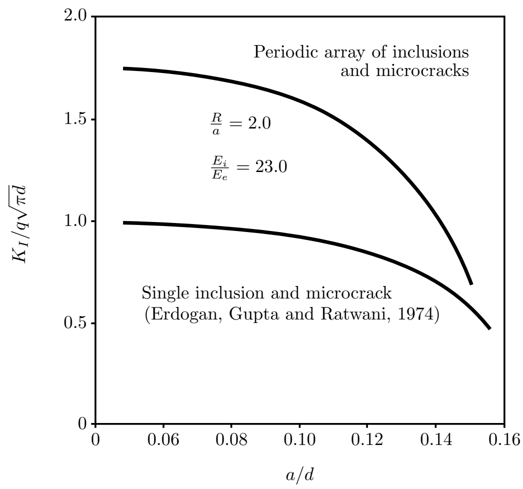

Figure 3: Comparison of results for the normalized stress intensity factor $K_I/(q\sqrt{\pi a})$ corresponding to a single microcrack interacting with single inclusion and the periodic array of microcracks interacting with the array of inclusions.

and it is bounded on the closed interval $s \in [-1, 1]$. It describes the interaction between the microcracks and inclusions and in the absence of inclusions $g(1)$ verges into the expression $q(1 + \kappa_e)/(2\mu_e)$, where q denotes the remote loading. This recover the relation for the stress intensity factor for an infinite row of collinear cracks. The equation (1) can be presented in the form of the relation of the normalized stress intensity factor, $K_I/(q\sqrt{\pi d})$, versus the normalized microcrack length, a/d, for several values of R/d and the elastic moduli ratio, see Fig. 2. These graphs show an expected result that elastically softer/stiffer inclusions amplify/attenuate the stress intensity comparing to the homogeneous case. It is also a matter of interest to compare the stress intensity factor for the single crack interacting with a single inclusion lying in the plane of crack, see Erdogan, Gupta and Ratwani [25], with the stress intensity factor for the periodic array of cracks and inclusions. A comparison of solutions calculated for the ratio of the inclusion radius and the microcrack half-length $R/a = 2$ and for the ratio of elastic moduli of inclusions and matrix $E_i/E_e = 23$ is shown in Fig. 3.

The previous problem of the array of the microcracks and inclusion can be generalized to the case, when the microcrack tips impinge the inclusion. Then the singularity exponent differs from the value 1/2 and must be evaluated from the boundary and material conditions prevailing in the vicinity of the stress singularity. Also in this case the singular stress field in the vicinity of a singular point exhibits the asymptotic behavior $\sigma_{ij} \sim r^{\delta-1}$, where r is the distance from the singular point and δ is called a characteristic eigenvalue of the singularity which acquire a value from the interval $(0, 1)$. It can be also generally complex, whereas the real part is from interval $(0, 1)$. As the other example of the general stress concentrator can be considered not only the crack, but also the notch with tip on the interface of two dissimilar materials, interface crack and generally a junction of several materials, see Fig. 4.

Within the framework of the linear-elastic fracture mechanics, the stress field in the vicinity of the general stress concentrator is possible to write, for the general case of loading, in form, see Williams [17],

$$\sigma_{ij} = H_1 r^{\delta_1 - 1} f_{ij1}(\varphi) + H_2 r^{\delta_2 - 1} f_{ij2}(\varphi) + T\delta_{i1}\delta_{j1} + O(r^{\delta}), \tag{2}$$

where the amplitude of the first and second term (singular terms) are called *Generalized Stress Intensity Factor*s (GSIFs). The H_1 corresponds to the generalized stress intensity factor of a

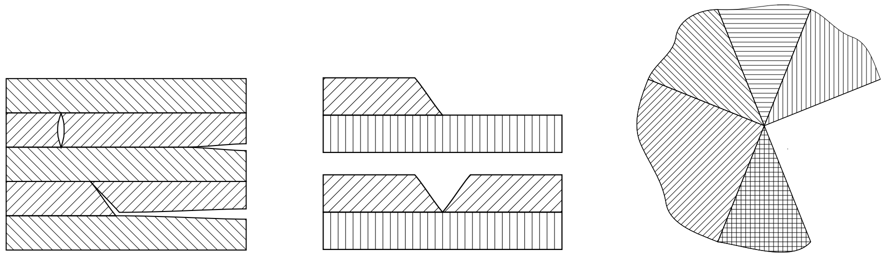

Figure 4: Different types of the general stress concentrators – crack terminating at the interface of two dissimilar materials, interface crack, notch and V-notch and a general multimaterial wedge.

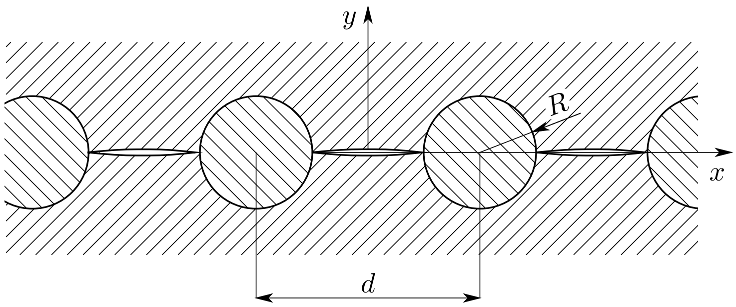

Figure 5: The periodic array of the inclusions and the microcracks with their tips impinging the inclusions.

stronger singularity than H_2. Generally, the functions $f_{ijk}(\varphi)$ depend on the geometry and material characteristics of the stress concentrator. Symbol T is associated with the T-stress and $O(r^\delta)$ are the higher order terms which are negligible in comparison with the previous ones for $r \to 0$.

The values of the singularity exponents of the general stress concentrators must be evaluated numerically, but for the case of isotropic materials, the formula giving the exponent values is well known and can be found in the closed form, see e.g. Hills, Kelly, Dai and Korsunsky [28]. The evaluation of the stress singularity exponent is the essential step of the general stress concentrator analysis. Its knowledge allows the formula (1) to be rewritten to its generalized form for the above discussed problem of the arrays of microcracks and inclusions, when the microcrack tips terminate at the inclusions, see Fig. 5,

$$H_I = (2\pi)^{1/2-\lambda}\,\mu^*\left[d\tan\left(\frac{\pi a}{d}\right)\right]^\lambda g(1), \tag{3}$$

where μ^* and Dundurs' parameters α and β can be found in e.g. Hills, Kelly, Dai and Korsunsky [28], and H_I is the generalized crack tip stress intensity factor. From the equation (2) follows, that $\delta_1 = 1 - \lambda$.

Because of the difficult physical interpretation of generalized stress intensity factor H_I by reason of its different physical dimension than $\mathrm{MPa}\cdot\mathrm{m}^{1/2}$, the standard fracture criterion of the linear fracture mechanics cannot be used. To be able to quantify the effect of the inclusions on the propagation of the microcracks, it is considered the stability criterion which is related to the average stress calculated across the distance d^* from the microcracks tips, where d^* is the characteristic dimension of the fractured inclusions. The numerical values of the normalized applied loading q^c/q^c_{hom} is displayed in Fig. 6. The q^c_{hom} is the critical applied loading of the array of the collinear microcracks and the absence of the inclusions.

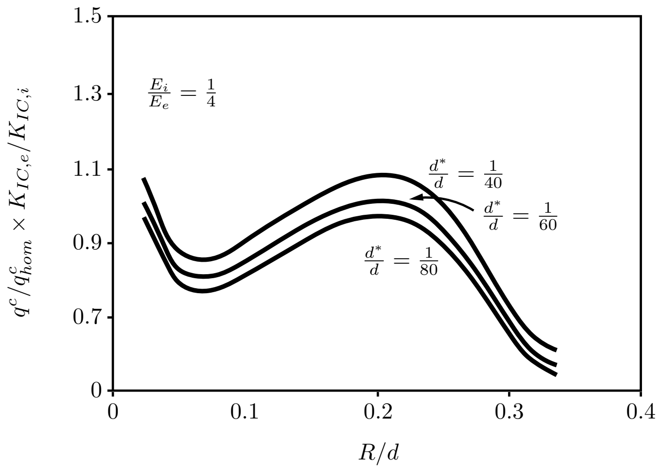

Figure 6: Normalized critical applied stress for the propagation of the microcracks. Parameters $K_{IC,i}$ and $K_{IC,e}$, respectively, are the fracture toughnesses of the inclusions and matrix, respectively.

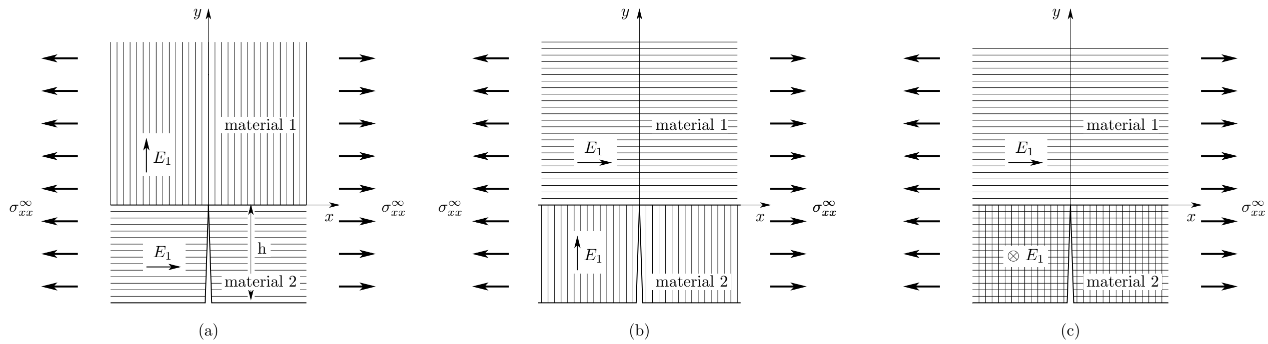

Figure 7: Cracked and loaded specimens made of two layers of composite such as Graphite/Epoxy T300/5208 system. There are indicated three different mutual orientations of layers inside the shaded domains.

The article [a2] deals with the solution and description of the stress intensity factor and the critical applied loading as it is discussed above. So do the articles [a4]–[a8], [a10], [a11], but the results are applied to the problems of the composite fracture, e.g. crack path stability for a periodic array of cracks and inclusions, the crack tunneling in the composites composed by the brittle matrix reinforced by the tougher fibers and the weakening effect of the microcracking ahead of the main crack.

Because the significant part of composite materials exhibit the anisotropic behavior, the other important area of the application of the linear fracture mechanics is the stress concentrator analysis in the anisotropic elasticity. It might seen that the anisotropy makes all the previous calculations more complicated, but it is the half of the truth because of the existence of the Lekhnitskii, Eshelby and Stroh formalism (LES formalism), which is based on the complex function theory (complex potential theory) and gives an elegant and powerful tools for displacement and stress field description.

Supposing the configuration of the stress concentrator as it is displayed in Fig. 4, the third case. Providing a perfect bonding between two adjacent materials and application of appro-

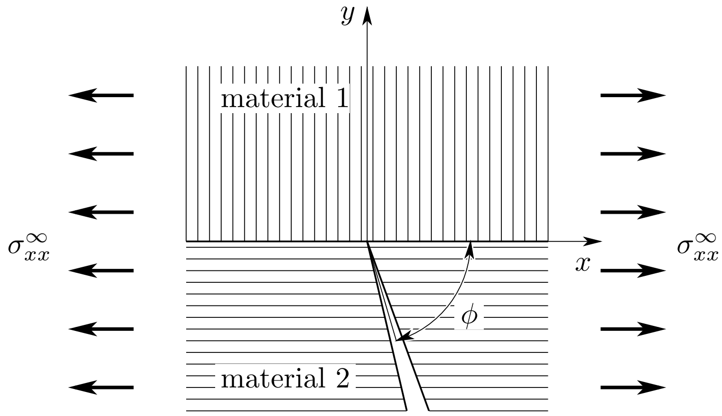

Figure 8: The bimaterial configuration with the crack situated at an arbitrary angle with respect to the bimaterial interface.

priate boundary conditions, one gets a system of $6N$ homogeneous linear equations, where N is the number of material wedges, Desmorat and Leckie [29]. The number of homogeneous linear equations can be always reduced to the system of three linear dependent homogeneous equations and this system is shortly possible to write in the form

$$\boldsymbol{K}(\delta)\boldsymbol{v} = 0, \tag{4}$$

where for non trivial solution the determinant of the system matrix $\boldsymbol{K}$ have to be zero. From this condition one gets a non-linear equation whose roots are the searched characteristic singularity eigenvalues δ. The real part of the least root from interval $(0, 1)$ defines a singularity exponent $\delta_1 - 1$, see (2). It should be pointed out, that there exist many other method providing the evaluation of the singularity exponents, see [a13], [a23], [a25], [a26], [a33] or Ševeček [30].

As an illustration of the generalized stress intensity factor application to the stress concentrators in anisotropic, especially in the orthotropic material, can be taken the results published in the papers [a23] and [a25]. There was studied the specimen, which was made of two layers of composite such as Graphite/Epoxy T300/5208 system. The loading of the specimen and material variation of the layers is displayed in the Fig. 7. There are considered three different mutual orientations of layers of material 1 and 2 in the Fig. 7 with the axis of material symmetry either parallel or perpendicular to the bimaterial interface. For each of considered configuration, the eigenvalue problem (4) gives the pair of quantities, δ and $\boldsymbol{v}$. This quantities pertain to the real solution, but it should be pointed out, that the eigenvalue problem (4) gives also the pair of quantities, $-\delta$ and $\boldsymbol{w}$, pertaining to the so-called *auxiliary solution*, whose knowledge is essential for the evaluation of the generalized stress intensity factor by the Ψ-integral method offering an effective and rather accurate way to generalized stress intensity factor evaluation.

The problem of the perpendicular crack to the bimaterial interface can be generalized to the crack inclined with respect to the interface, see Fig. 8 and [a32], [a33] and Ševeček [30]. In the same way as in the previous crack configuration, the characteristic eigenvalues of the singularity δ_1 and δ_2 are calculated using the LES method. The whole stress and displacement field is described by the two singular terms characterized by the two pairs of stress singularity exponent $1 - \delta_i$ and corresponding generalized stress intensity factors H_i, $(i = 1, 2)$. The results are displayed in Fig. 9 for Dundurs' parameter $\beta = 0$, material parameters $\lambda_1 = 0.1$, $\rho_1 = 2$, $\lambda_2 = 1$, $\rho_2 = 1$, $E_2 = 60\,000$ MPa, $\nu_2 = 0.238$ and several values of Dundurs' parameter

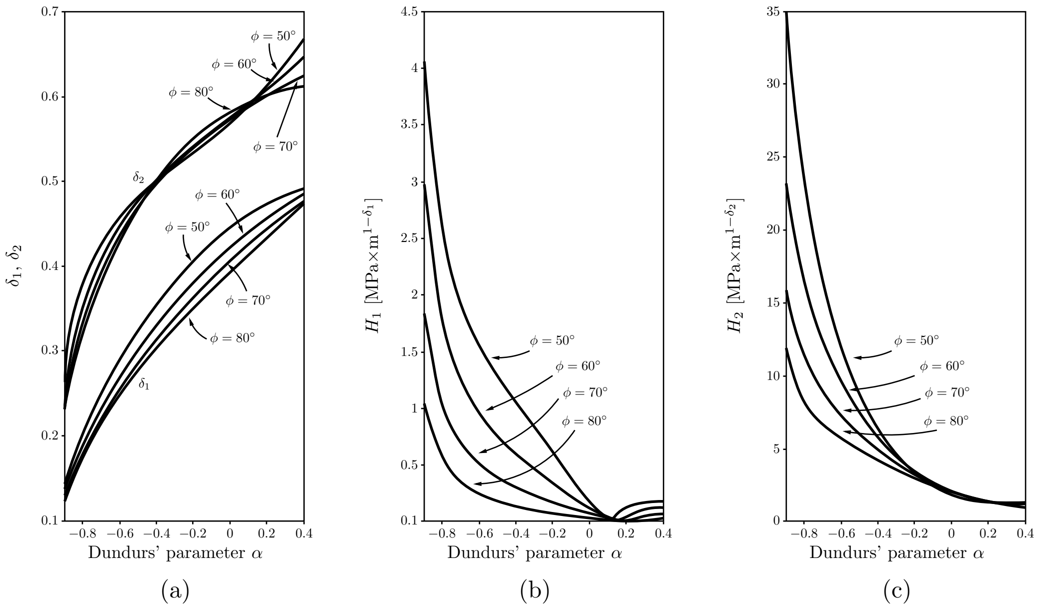

Figure 9: Variation of the eigenvalues δ_1, δ_2 and of the generalized stress intensity factors H_1, H_2 with Dundurs' parameter α for several angles ϕ of the crack inclination with respect to the bimaterial interface (Dundurs' parameter $\beta = 0$).

α and angles ϕ.

The next generalization of the problems discussed above is the study of the orthotropic bimaterial notch. As an illustration of this bimaterial configuration can be taken the results given in [a36], where it is studied the rectangular bimaterial orthotropic notch, see the Fig. 10.

The last example of the generalization of the stress intensity factor is the case of the bridged crack impinging the bimaterial interface discussed in [a28]. Assuming the crack initiated from the surface defect and extended through the surface layer of the thickness h, see Fig. 11. The investigation of the toughening mechanism can be specified as the finding the local generalized stress intensity factor H_{tip} related to the remote applied stress intensity factor H_{appl} and the generalized bridging stress intensity factor H_{br} by the formula

$$H_{tip} = H_{appl} - H_{br}. \tag{5}$$

The problem is solved using two methods. The first one consists in the application of the just

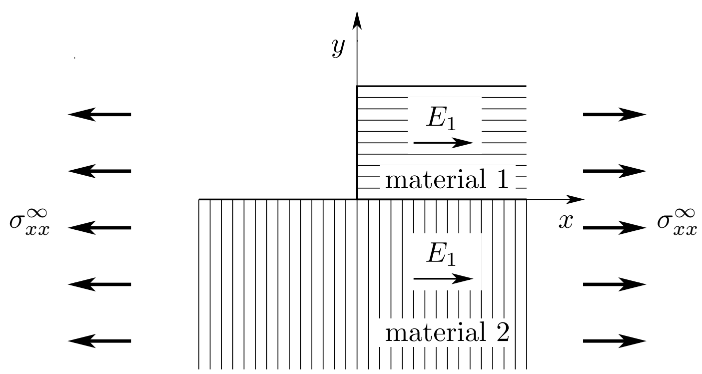

Figure 10: The remote loading and geometrical configuration of the bimaterial orthotropic notch.

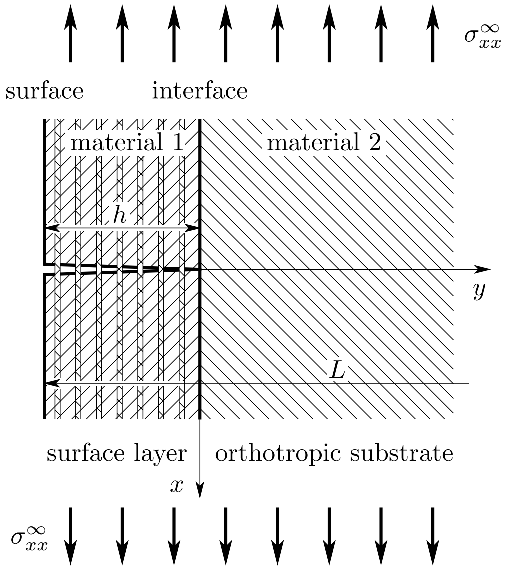

Figure 11: The remote loading and geometrical configuration of the bridged crack impinging the bimaterial interface.

mentioned Ψ-integral, but with simple modification, which is based on the incorporation of a pair of body forces acting on the crack faces at the general point. This pair of the body forces allows the weight functions $W(y, h)$ to be set up and used further to calculate the generalized bridging stress intensity factor, H_{br}.

There is evaluated the weight function in the dimensionless form $W \cdot h^{1-\delta}$ against the dimensionless distance from the crack tip $-y/h$ in the Fig. 12. The specification of material combination of the layers 1 and 2 corresponds to the composite such as FP/A1 system with elastic constants $E_L = 225$ GPa, $E_T = 150$ GPa, $G_L = 58$ GPa, $\nu_L = 0.28$, where the fibers of the material of the substrate (material 2) are parallel with y axis. It should be noted, that this bimaterial configuration leads to the value of the stress singularity exponent $\delta = \delta_1 = 0.672$. The second method used for the evaluation of the local generalized stress intensity factor H_{br} is the crack modeling by the distributed dislocation technique. The advantage of this method is its accuracy, even though it is able to give the numerical results only for the first stage of the loading, when the broken fibers are not massively pulled out from the matrix because there are a certain numerical problems for the subsequent stage of loading. Whether the H_{br} is evaluated by the weight function method or distributed dislocation technique, it necessary to know the bridging stress distribution, $\hat{\sigma}_{br}$, along the crack faces, see [a28]. As an illustration, there are the plots of the $\hat{\sigma}_{br}$ for several values of the Weibull modulus, m, the frictional shear stress between the matrix and fibers, τ, and strength of the fibers, σ_{0f}, in the Fig. 13. Having the bridged stress dependence on the crack opening displacement the generalized bridging stress intensity factor, H_{br}, can be evaluated. The results of these calculations are presented in Fig. 14, where the remote, bridging, and local generalized stress intensity factors are plotted as functions of the applied tensile loading, σ_0, for several values of the Weibull modulus, m. It is a matter of interest to compare the calculations based upon the weight function method with the results obtained using the distribution dislocation technique in the first stage of loading. It is seen, that the results obtained using the distribution dislocation technique are in a good accordance with the results received from the calculations based on the weight function method.

The T-stress also has a significant influence on crack initiation angles in brittle fracture,

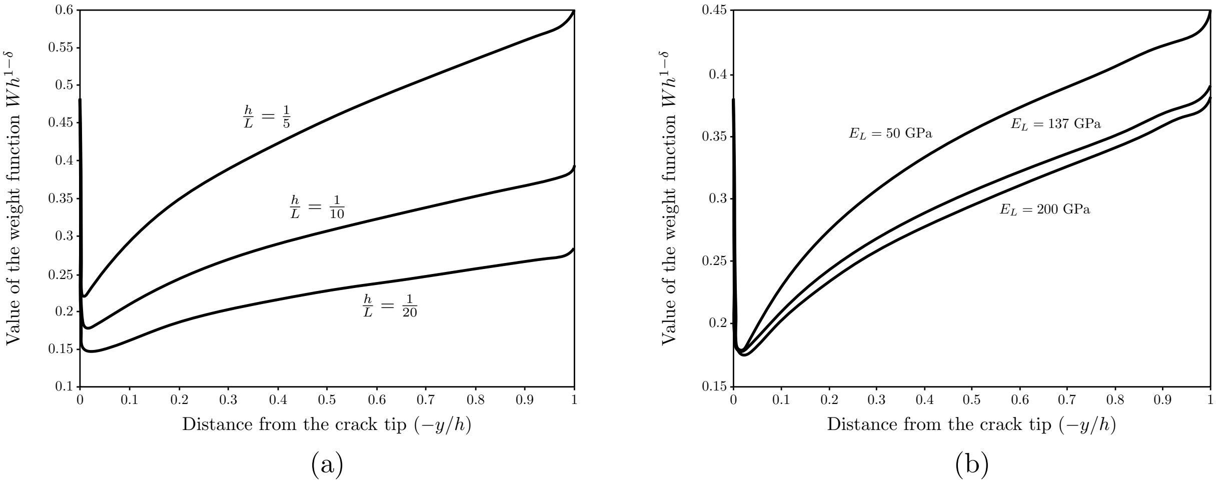

Figure 12: Bimaterial normalized weight function against the dimensionless distance from the crack tip for several values of (a) the ratio h/L, (b) the longitudinal modulus E_L.

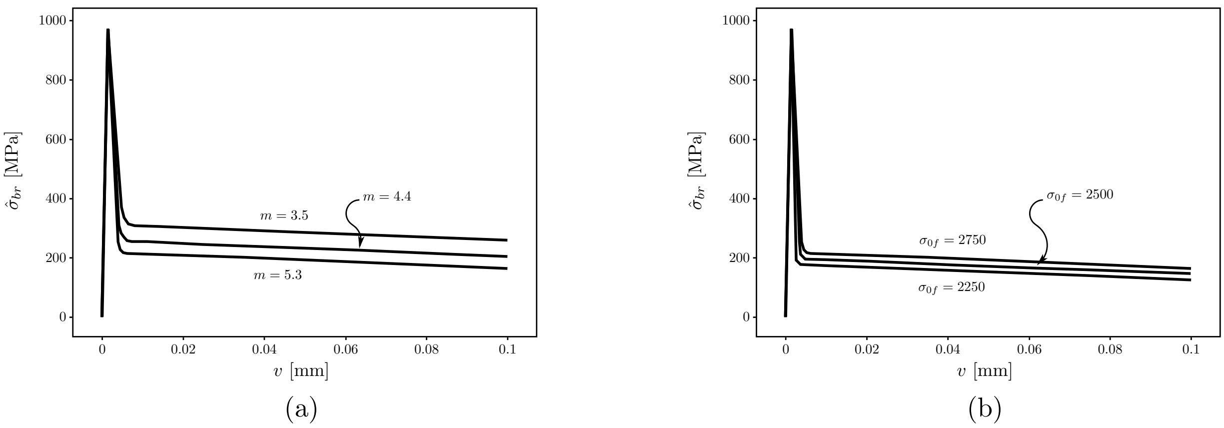

Figure 13: Bridging stress $\hat{\sigma}_{br}$ as a function of $v = \delta(x)/2$ for several values of (a) the Weibull modulus, m, $\tau = 6$ MPa, $\sigma_{0f} = 2750$ MPa and (b) the characteristic fiber strength, σ_{0f}, $m = 5.3$, $\tau = 6$ MPa.

Melin [32]. In general, numerical determination of T-stresses requires careful handling, because of their location in the vicinity of the singular points. As suggested by Broberg [33], the T-stress can also be determined using dislocation arrays.

As an typical example of the problem, where the T-stress should be studied, can be taken the configuration of the crack given in the Fig. 7a. The formula for the T-stress evaluation can be written in the following form,

$$T = \lim_{t \to 1^-} \left[\mathrm{Re} \left\{ \frac{L_{21}^{II} M_{11}^{II}}{p_1^{II}} + \frac{L_{22}^{II} M_{21}^{II}}{p_2^{II}} \right\} \left(\frac{h}{2} \sigma_{xx}^{ns}(t) + \sigma_{xx}^{appl}(t) \right) + \frac{h}{2} \sigma_{yy}^{ns} \right], \tag{6}$$

where the matrix $\boldsymbol{M}$ is the inverse of $\boldsymbol{L}$ defined in [a25], [a26], p_i are the eigenvalues of the material and the index I or II, respectively, stands for material 1 or 2, respectively, see [a25], [a26]. The $\sigma_{xx}^{appl}(y)$ corresponds to the stress along the negative y axis in the material 2 without the crack and it is determined using the FEM. Stresses σ_{ii}^{ns} correspond to nonsingular part of the stress field around the crack involving the interaction of the free surface, coating and

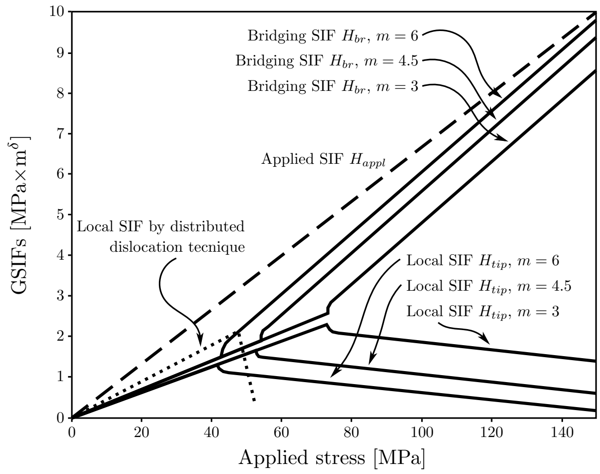

Figure 14: Remote, bridging, and local generalized stress intensity factors plotted as functions of the applied tensile loading σ_0 for a several values of the Weibull modulus.

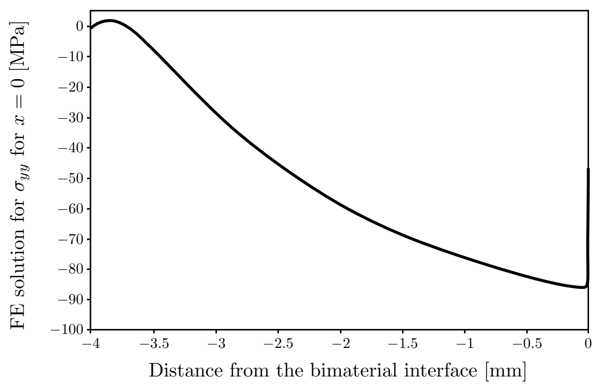

Figure 15: Finite element solution for $\sigma_{yy}(y)$ along the crack.

substrate. As an example, that the finite element analysis do not offer practicable way to express the T-stress, is shown in the Fig. 15. The T-stress is the limit $\sigma_{yy}(y)|_{y\to 0^-}$ and it is clear that this value is difficult to estimate from the figure since the curve exhibits a turning point very close to the crack tip and sharply increases behind this point. Thus a rough estimate of the T-stress is about of -50 MPa.

As it is discussed in the previous text, the first two elements H_1 and H_2 of the Williams' asymptotic expansion (2) can be evaluated using the numerical-analytical procedure based on the so-called Ψ-integral. The necessary condition for the Ψ-integral application to the stress singularity problem is the knowledge of the auxiliary solution arising from the eigenvalue problem (4). Generally, it is possible to express any coefficient of the Williams' asymptotic expansion if the corresponding auxiliary solution is available. An example of the evaluation of the T-stress by this way can be found in [a34]. The schema of the studied problem is in Fig. 16 and the calculated T-stress is displayed as a function of the spacing to layer thickness ratio for the mechanical loading in the Fig. 17.

It should be pointed out, that there are other results and mainly the procedure descriptions being related to the T-stress in the papers [a25], [a26], [a33] and [a34].

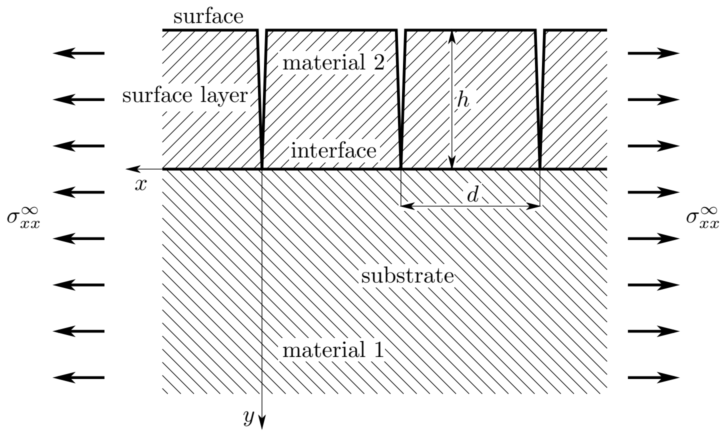

Figure 16: The scheme of an array of periodically distributed edge cracks.

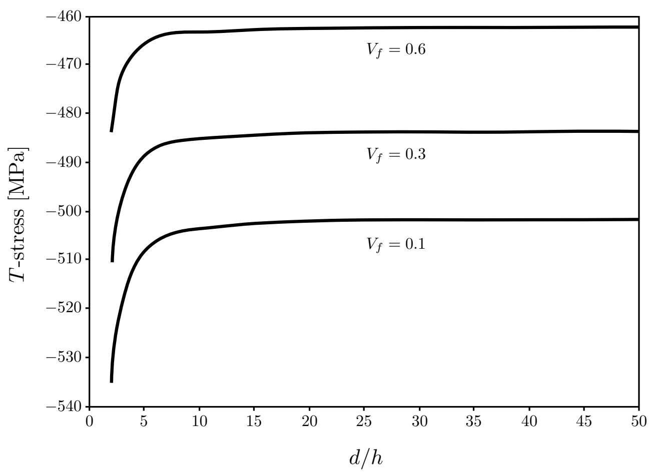

Figure 17: The T-stress as a function of the spacing to a layer thickness ratio for the case of mechanical loading of $\sigma_{xx}^{\infty} = 1000$ MPa and several values of the fiber volume fraction V_f.

2 An application of the theory of the complex potentials to some problems of the fracture mechanics

The purpose for the using of the complex potentials in the problems are follows.

- To get the description of the stress and displacement field in the domain with the absence of the crack and under the remote loading. The knowledge of this stress field is necessary for the later crack modeling via the method of the continuously distributed dislocations. If this solution is not available, it must be compensated by some numerical solution, e.g. by finite element method.
- To construct the Green's function corresponding to the solved problem. Also the knowledge of the Green's function is essential for the crack modeling by the just mentioned continuously distributed dislocation technique, but the Green's function construction is rather difficult and it is often conditioned by geometry simplification of the domain, on which the problem is solved.
- The last purpose of the complex potential using is the quantification of the stress sin-

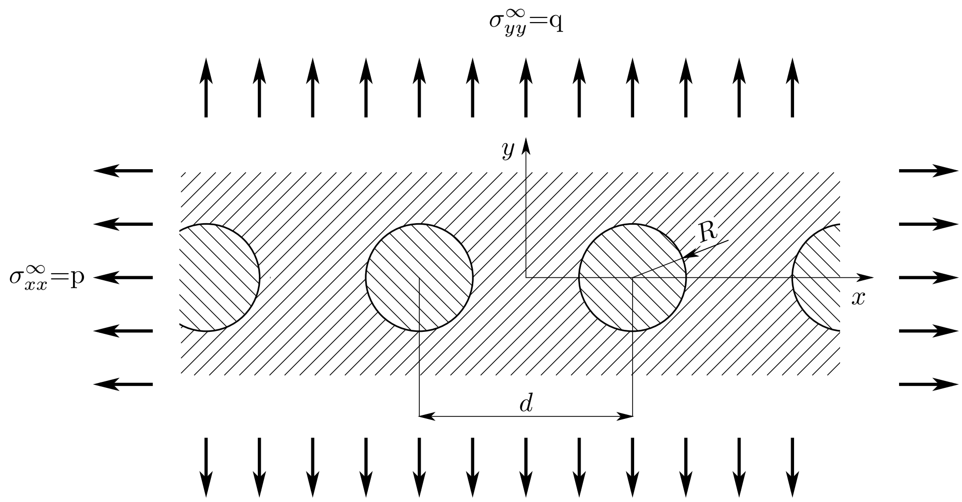

Figure 18: The periodic array of isotropic inclusions in the isotropic matrix under the remote loading $\sigma_{xx}^{\infty} = p$, $\sigma_{yy}^{\infty} = q$.

gularity at a crack tip impinging the bimaterial interface or at the tip of the bimaterial notch. The additional result of this stress singularity analysis is the receiving of the regular and so-called auxiliary solutions which are further introduced into the later discussed Ψ- integral allowing e.g. evaluation of the generalized stress intensity factor.

The first of the above mentioned aims is performed in the papers [a2]–[a11]. The problem is assumed as the plane strain one for an infinite periodic array of circular inclusions loaded at infinity by stresses $\sigma_{yy}^{\infty} = q$ and $\sigma_{xx}^{\infty} = p$, see Fig. 18. The inclusions of radius R lie along the x axis with their centers at points

$$x = \frac{d}{2} \pm md, \tag{7}$$

where d denotes the distance between inclusion centers and $m = 0, 1, 2, \ldots$ The problem was originally solved in Kosmodamiansky [35], but there is used slightly modified method of solution in [a2]–[a11]. Assuming a perfect adhesion between the inclusions and the infinite medium, then the continuity of displacement and traction at the inclusions and matrix interfaces γ requires

$$u_e + \mathrm{i}v_e = u_i + \mathrm{i}v_i,\ \sigma_{rr,e} + \mathrm{i}\sigma_{r\theta,e} = \sigma_{rr,i} + \mathrm{i}\sigma_{r\theta,i} \quad \text{for } z \in \gamma, \tag{8}$$

where the subscript e and i refers to the matrix and the inclusions, respectively. The solution is found using the Muskhelishvili's potentials $\varphi(z)$ and $\psi(z)$ in the form of series composed of the basis functions $(z \pm d/2)^k$, $k = 0, 1, \pm 2, \ldots$ According the Galerkin-Bubnov method one obtains a system of linear equations from which the coefficients $\alpha_{i,n}$ and $\beta_{i,n}$ of the series for the inclusions and matrix may be determined. As an numerical example, the normalized stress σ_{yy} appearing in matrix between the inclusions for several values of the Young modulus ratio E_i/E_e is shown in Fig. 19, see [a10].

For successful crack modeling, the knowledge of the corresponding Green's function is necessary. Again, there is one way of the Green's function construction presented in the papers [a2]–[a11]. Assume again an infinite periodic array of circular inclusions, but instead of the external loading, a periodic array of edge dislocations with Burgers vector $\boldsymbol{b} = (0, \pm b_y)$ is introduced into the matrix along the x axis in such a way that the sign of Burgers vector

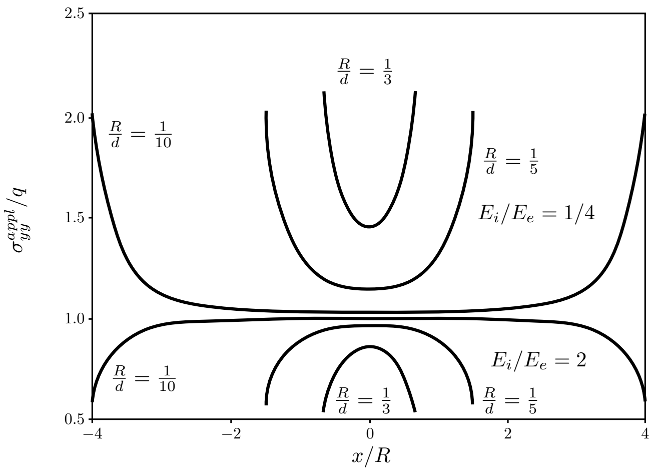

Figure 19: Normalized stress σ_{yy}^{appl}/q acting between inclusions in uncracked body for $R/d =$ 0.1, 0.2, 0.3 and $E_i/E_e = 1/4$ (upper curves), 2/1 (lower curves) as a function of dimensionless distance x/R measured from point between inclusions.

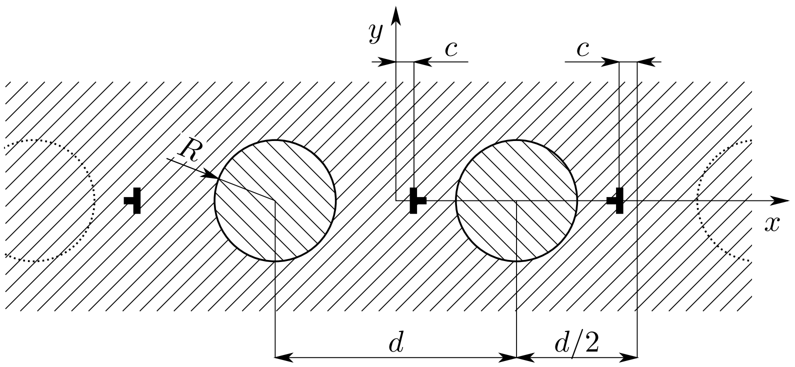

Figure 20: Periodic array of inclusions and dislocations. Shaded inclusions and dislocations at the points $x = c$ and $x = d - c$ stand for the unit cell.

is changing periodically, see Fig. 20. The procedure for the finding the solution is the same as in the previous case of the periodic array of inclusions under the remote loading which is replaced by the presence of the array of dislocations, see [a10]. It should be pointed out, that they are the sum of the solutions of the one inclusion and one dislocation interaction given in Dundurs and Mura [40].

The next examples of the theory of the complex potentials application such as the Green's function serving for the modeling of the crack in the orthotropic bimaterial composed of thin layer and semi-infinite substrate can be found in e.g. [a28], [a25] or [a26]. The other aim of the complex potential theory used in the papers [a13]–[a34] is its elegant application to the eigenvalue problem resulting from the analysis of the stresses prevailing in the region enclosing the crack tip or bimaterial orthotropic notch tip, see [a36].

3 The distributed dislocation technique in the crack problems

The distributed dislocation technique leads to the solution of singular integral equation with the singular simple Cauchy kernel. The solution of the compiled singular integral equation is the dislocation density $B_y(x)$, whose knowledge allows the evaluation of the basic quantities such as opening displacement, $\delta(x)$, and stress intensity factor, K_I. The aim of the research is the interaction of the cracks with the other parts of the body such as inclusions, free boundaries and interfaces. The problems presented e.g. in the articles [a2]–[a11], [a13], [a28] and [a25] lead to the solution of the sophisticated integral equation, which has to be solved numerically. There are a number of effective numerical procedures for handling singular equations with Cauchy kernels such as the procedures based on the Gauss-Chebyshev quadrature, Erdogan and Gupta [49], Erdogan, Gupta and Cook [50].

As an example of the continuously distributed dislocation technique application and the subsequent solution of the appropriate singular integral equation can be found in [a28]. The problem is discussed in the previous chapters and the crack within the bimaterial is shown in the Fig. 11. The distribution of the dislocations along the crack line, the introduction of the traction induced by the crack bridging zone and the utilizing of the so-called Bueckner's principle leads to the following singular integral equation

$$\sigma_{xx}^{appl}(t) + \widehat{\sigma}_{br}\left(\delta(t)\right) + \frac{1}{\pi}\mathrm{Re}\left\{\sum_{\beta=1}^{2} L_{1\beta}^{II}\sum_{\alpha=1}^{2} M_{\beta\alpha}^{II}\left(B_{\alpha 1}^{II}\right)^{-1}\right\}\int_{-1}^{1}\frac{B_x(s)}{s-t}\mathrm{d}s + \int_{-1}^{1} B_x(s)K_{xx}(t,s)\mathrm{d}s = 0, \tag{9}$$

where

$$K_{xx}(t,s) = \sum_{n=1}^{N_k}\frac{k_{1,n}}{k_{2,n}s - k_{3,n}t - k_{4,n}} \tag{10}$$

is the regular kernel describing the interaction of a dislocation with the bimaterial interface and with the free surface as well, see [a28]. Since the material interface and the crack plane correspond to the material symmetry planes and the specimen is subjected to a simple tensile loading conditions, the Burgers vector component b_y is equal to zero. The $\sigma_{xx}^{appl}(t)$ denotes the negated stresses in $x = 0$ produced by the given boundary loads acting on a specimen, but without crack. The $\widehat{\sigma}_{br}\left(\delta(t)\right)$ is the bridging stress shown in the Fig. 13, see [a28], which depends on the crack opening displacement $\delta(y)$ and must be calculated by the iterative way. The dislocation density is sought in the form

$$B_x(s) = (1-s)^{-\lambda}(1+s)^{\lambda}g(s), \tag{11}$$

where λ is the stress singularity exponent and $g(s)$ is the bounded function. The chosen form of $B_x(s)$ given by (11) allows one to express the integral containing the regular kernel $K_{xx}(t,s)$ in the closed form by integrating each component of the truncated series (10) using the theory of the curve complex integrals developed by Muskhelishvili, [18]. The finally expression of the integral is rather complicated and can be found in [a28]. The integral equation (9) is solved using the Gauss-Jacobi quadrature implying that the function $g(s)$ is sought in the form of linear combination of Jacobi polynomials

$$g(s) = \sum_{n=0}^{\infty} c_n P_n^{(-\lambda,\lambda)}(s) \cong \sum_{n=0}^{N_B} c_n P_n^{(-\lambda,\lambda)}(s). \tag{12}$$

A singular integral in (9) then may be expressed in the closed form, see Erdogan, Gupta and Cook [50], Kaya and Erdogan [51], depending on the collocation points t_i, which are chosen so, that

$$t_i = \cos\left(\frac{\pi}{2}\frac{2i+1}{N_B}\right) - 1, \tag{13}$$

where $i = 0, 1, \ldots, N_B - 1$ and N_B is the degree of the Burgers vector approximation. The substitution of the integrated regular kernel and the expresion of the singular integral into the singular integral equation (9) leads to the algebraic equations in the unknowns c_n. Their evaluation gives the dislocation density function $B_x(s)$ and consequently the value of the crack opening displacement. This allows the bridging stress $\widehat{\sigma}_{br}\left(\delta(t)\right)$ to be corrected and the following recalculation of the coefficients c_n can be made. The convergence any of these parameters should be appeared and the procedure continue until the required accuracy is achieved. The knowledge of the interpolation of the Burgers vector density (11) allows the evaluation of the generalized stress intensity factor, whose formula is given in [a28]. Mostly it is impossible to integrate the regular kernel appearing in the integral equation resulting from the problems similar to that one discussed above. In these cases, the integrals must be evaluated numerically or may be approximated by the set of suitable functions, see [a10].

The distributed dislocation technique can also be applied to the problem of the determination of the T-stress, [a25]. The distributed dislocation technique also offers possibility of the stress singularity exponent evaluation for the cracks impinging the bimaterial interface. This problem is not discussed here, but also this application of the distributed dislocation technique shows the effectiveness of this method in the crack problems. The basics of the method can be found in Gupta, Argon and Suo [31] and its applications including also the construction of the regular and auxiliary solutions of the solved problems. These allow the expression of the generalized intensity factors or T-stresses via the Ψ-integral treated in e.g. [a13], [a25], [a26], [a34] and Ševeček [30].

4 The two-state integrals

Except the distributed dislocation technique, there are next several approaches for the amplitude calculation of singular and the other terms in the Williams asymptotic expansion (2). One of the simplest to evaluate the generalized stress intensity factor is based on the comparison of numerical calculations of the stress (or displacement) field in front of the crack tip (e.g. by finite element method) with the appropriate analytical expressions for stresses or displacements. The generalized stress intensity factor is then extracted for $r \to 0$, e.g. [a31], Náhlík [52]. Another effective method, which can be used for the generalized stress intensity factor calculation, eventually also for the T-stress calculation, is based on the method of two-state integrals in combination with finite element method. The two-state integrals, which are path independent, are based on the J-integral, Gross and Seelig [57], Chang and Wu [55], or M-integral, Gröger [58]. However, the J-integral cannot be applied for the calculation of generalized stress intensity factors in the cases of V-notches or other general stress concentrators, because is not path independent, and even though the M-integral does not lose its path independence in these cases, it is seem to be very useful to introduce another two-state integral, so-called Ψ-integral, see Ševeček [30], Hwu [34]. The detailed information about the two states integrals can be found in, e.g. Akisanya and Fleck [54], Desmorat and Leckie [29], Chang and Wu [55], Im and Kim [56].

The method of Ψ-integral enables to determine the local stress field parameters in the vicinity of the crack or notch tip using the deformation and the stress field in the remote

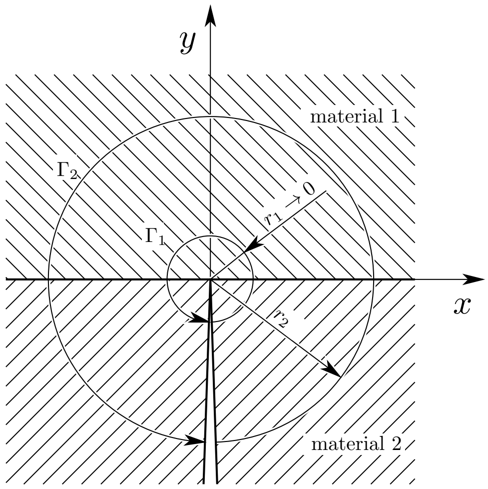

Figure 21: Integration paths surrounding the singular point.

points, where the numerical results obtained e.g. using the finite element analysis are more accurate. This method which turned out to be very efficient is an implication of the Betti's reciprocity theorem. The reciprocal theorem states that in the absence of the body forces and residual stresses the following integral is path independent

$$\Psi(\boldsymbol{U},\boldsymbol{V}) = \int\limits_{\Gamma} \left[\sigma_{ij}(\boldsymbol{U}) n_i V_j - \sigma_{ij}(\boldsymbol{V}) n_i U_j\right] \mathrm{d}s, \tag{14}$$

where Γ is any contour surrounding the crack tip and $\boldsymbol{U}$, $\boldsymbol{V}$ are two admissible displacement fields, see Fig. 21. The asymptotic expansion of the displacements $\boldsymbol{U}(r,\theta)$ for the crack perpendicular to the interface of the orthotropic materials, see Fig. 7, is possible to write in the following form

$$\boldsymbol{U}(r,\theta) = H_1 r^{\delta_1} \boldsymbol{u}_1(\theta) + H_2 r^{\delta_2} \boldsymbol{u}_2(\theta) + T r \boldsymbol{u}_3(\theta) + \ldots = \sum_{i=0}^{\infty} k_i r^{\delta_i} \boldsymbol{u}_i(\theta), \tag{15}$$

where H_1 and H_2 are the generalized stress intensity factors, $\boldsymbol{u}_1(\theta)$ and $\boldsymbol{u}_2(\theta)$ are the angular distribution of the displacements corresponding to the singular terms in the stress asymptotic expansion and $\boldsymbol{u}_3(\theta)$ is the angular distribution of displacements for the T-stress. The application of the two-state integrals requires the knowledge of the so-called auxiliary (dual) solution $\boldsymbol{V}$ in the form of eigenfunctions of the appropriate singular problem. It can be proved, Papadakis and Babuska [43], that each regular solution of this eigenvalue problem generating the basis functions of (15), i.e. $r^{\delta_i}\boldsymbol{u}_i(\theta)$, is associated with the dual solution of the same eigenvalue problem, $r^{-\delta_i}\boldsymbol{u}_{-i}(\theta)$. Hence, considering the auxiliary solution $\boldsymbol{V}$ in the form $\boldsymbol{V}(r,\theta) = r^{-\delta_i}\boldsymbol{u}_{-i}(\theta)$ for $i = 1$, 2, 3, one can receive from (14) and (15) the generalized stress intensity factors, $H_1 = k_1$, $H_2 = k_2$ and T-stress, $T = k_3$ as follows

$$H_1 = \frac{\Psi\left(\boldsymbol{U}, r^{-\delta_1}\boldsymbol{u}_{-1}(\theta)\right)}{\Psi\left(r^{\delta_1}\boldsymbol{u}_1(\theta), r^{-\delta_1}\boldsymbol{u}_{-1}(\theta)\right)}, \tag{16}$$

$$H_2 = \frac{\Psi\left(\boldsymbol{U}, r^{-\delta_2}\boldsymbol{u}_{-2}(\theta)\right)}{\Psi\left(r^{\delta_2}\boldsymbol{u}_2(\theta), r^{-\delta_2}\boldsymbol{u}_{-2}(\theta)\right)}, \tag{17}$$

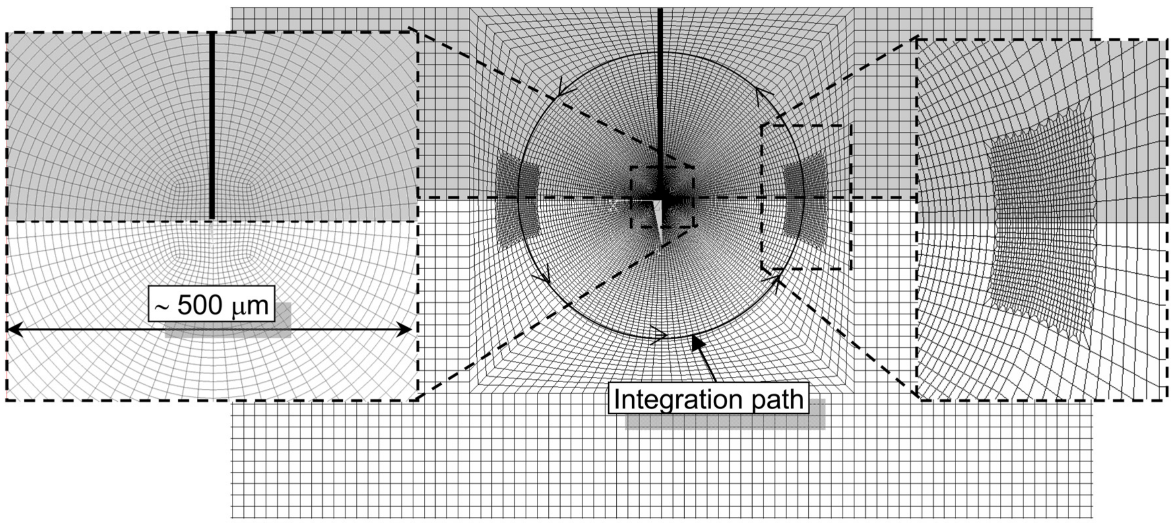

Figure 22: Example of the finite element mesh in the vicinity of the crack tip perpendicular to the bimaterial interface with detail of the mesh refinement along the integration path at interface crossing.

$$T = \frac{\Psi\left(\boldsymbol{U}, r^{-\delta_3}\boldsymbol{u}_{-3}(\theta)\right)}{\Psi\left(r^{\delta_3}\boldsymbol{u}_3(\theta), r^{-\delta_3}\boldsymbol{u}_{-3}(\theta)\right)}. \tag{18}$$

Observe, that the dual displacement fields, so-called extraction solutions, $r^{-\delta_i}\boldsymbol{u}_{-i}(\theta)$, are singular at the crack tip, hence they have unbounded energy near the crack tip and thus correspond to some concentrated sources at the crack tip. They are mathematical tools which allow extracting asymptotic coefficient terms from the complete exact solution $\boldsymbol{U}$. Since the exact solution $\boldsymbol{U}$ is not known, a finite element solution $\boldsymbol{U}^h$ can be used as an approximation for $\boldsymbol{U}$, see e.g. [a25], [a26].

The results of Ψ-integral concept applied to fracture problems are discussed in the previous chapter 1. There is treated only small note about some aspects of the numerical evaluation of the Ψ-integral in the following text.

The finite element analysis model for the $\boldsymbol{U}^h$ evaluation may show problems in the region where the integration path crosses the interface. Because of the discontinuity of some components of the stress tensor, the numerical errors can appear in the integration process. One possible way to reduce these errors is to introduce a finer mesh, Fig. 22. If the model is required to be simpler, to contain a smaller number of elements, the mesh refinement is not required and the errors without this refinement are relatively small and in some cases they are insignificant. Nevertheless, the study of this influence is recommended to perform before any larger computations and it is clear, that this manner of the error reduction is contrary to the main advantage of the Ψ-integral concept. To avoid this finite element model remeshing, one can use the interpolation of the stresses and displacements along the integration path via the splines. This interpolation should be done separately along the parts of the integration path divided by the crack faces and the bimaterial interface, where the path end-points belonging to the interface are excluded. After this process, a very efficient Romberg integration method can be applied to this interpolated function using any open or semi-open integration formulas, see Press, Teukolsky and Vetterling [64].

5 A conclusion

In this appearance, there were summarized several results published during the years 2003–2010 under prof. Michal Kotoul leadership or in conjunction with Jan Klusak from Institute of physics of materials, Academy of Sciences of the Czech Republic. These results are the product of the combination of the analytical and numerical methods, which were also discussed in the previous chapters. In this chapter, all the previous discussions are concluded.

It is clear, that the most important part of the treated research are the obtained results and their physical interpretation. The presented research is focused on the Irwin's concept of the linear fracture mechanics, i.e. the concept of the stress intensity factor, T-stress and energy release rate, and its application to the fracture of the composed materials. The first set of the results follows from the problem of the periodic array of collinear microcracks and inclusions. The problem was dealt as the isotropic plane problem and except the stress intensity factor and its generalization for various configurations of the cracks and inclusions, there were also discussed the critical applied loadings for the cracks impinging the inclusions interfaces, the crack path stability, the crack tunneling and the weakening effect of the microcracking ahead of the main crack.

The second set of the results is related to the bimaterials composed of the orthotropic materials, where one of this material is characterized by the finite thickness. The problems of the research discussed in the previous chapters concern the cracking of the finite thickness layer under the assumption of the crack or cracks impinging the bimaterial perfect bonded interface. The obtained results give the information about the stress singularity exponents, generalized stress intensity factors, T-stress, energy release rates and using the finite fracture mechanics, also about the next crack propagation into the substrate or along the interface.

As the third set of the results can be considered the generalization of the previous item to the case of the bimaterial notch. Also for this case there were discussed the obtained values of the stress singularity exponents and generalized stress intensity factors in this appearance.

The singular nature of the fracture mechanics problems solved requires a delicate handling. Apparently, the analytical method of the stress and displacement field description are irreplaceable. All of the obtained results discussed in this treatise stand on analytical basis and therefore the solved problems were considered as the plane ones. The first analytical method used to model the cracks in the research treated in the previous text was the distributed dislocation technique which leads to the solution of the singular integral equations. The application of this method is, in addition, conditioned by the knowledge of the Green's function corresponding to the solved problem. This Green's function can be received using the theory of the complex potentials in the plane isotropic or anisotropic elasticity. All these feature of the distributed dislocation technique were discussed in the previous chapters. Moreover, there was also discussed the weight function approaches applied into the related problems discussed in this work.

The application of the distributed dislocation technique has the limitations, particularly, this technique can be used in the problems with simple body geometry. To overcome this restriction, numerical methods, such as the finite element method, have to be used. But an inaccuracy near the stress concentrators may occur when the finite element analysis is applied to stress concentrator problems. Also it is impossible to evaluate all generalized stress intensity factors from the purely numerical solution. For this reason, it is convenient to introduce the method based on the combination of the two-state integrals with finite element method. However, the main contribution of the two-state integral method is its application to the establishing of the further crack growth criteria based on the finite fracture mechanics. The two-state integral method was discussed and applied to the significant part of the problems presented in this treatise, such as the crack inclined to the bimaterial interface or bimaterial notch.

Even though it is difficult to find different three words characterizing the present state of the fracture mechanics as the "finite", "element" and "method", the limits of this powerfull tool are given by the quality of the mesh of the studied domain. Therefore the analytical methods play an important role in the problems of the fracture mechanics. Especially, the theory of the complex potentials for the plane elasticity offers an effective instrument for the description of the stress and displacement field near the stress concentrators. Except the classical isotropic or anisotropic materials, the complex potentials can be used also for e.g. the piezoelectric materials or thermoelasticity.

The finite element method, fortunately, is not only one numerical method allowing researches or engineers to solve the elasticity problems defined on the general domains. A significant alternative to the finite element method is the boundary element method and its variations such as the hybrid boundary element method and dual reciprocity boundary element method. Moreover, there are the ways connecting the boundary element method and distributed dislocation technique.

All these mentioned matters give the valid arguments for the application and combination all these methods in the further research of the particular problems of the fracture mechanics.

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Abstract

In the problems of the fracture mechanics, where the cracks are relatively short with respect to the geometrical characteristic feature of the solved plane problem, it is practicable to represent the geometry in an idealized way. This idealization allows one to use the analytical methods based on the theory of the complex potentials, which subsequently enables to utilize the crack modeling by the distributed dislocation technique. An advantage of this procedure is its accuracy particularly significant in the crack problems of the bimaterial interface. If the idealization of the geometry cannot be applied and the finite element method has to be employed, it is appropriate to combine its numerical results with the Ψ-integral method, which augments the accuracy of the obtained numerical results removing the affected requirements to the finite element mesh. All these methods can be effectively introduced and used in the problems of the stress concentrators at the bimaterial interfaces between the isotropic and orthotropic materials. Their application enables to obtain all characteristic describing the stress concentrators behavior, especially the generalized stress intensity factor, energy release rate and T-stress.