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Linear Matrix Differential Equation with Delay

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LINEAR MATRIX DIFFERENTIAL EQUATION WITH DELAY

LINEÁRNÍ MATICOVÉ DIFERENCIÁLNÍ ROVNICE SE ZPOŽDĚNÍM

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Contents

1 Introduction

Individual results for functional-differential equations were obtained more than 250 years ago, and systematic development of the theory of such equations began only in the last 90 years. Before this time there were thousands of articles and several books devoted to the study and application of functional-differential equations. However, all these studies consider separate sections of the theory and its applications (the exception is well-known book Elsgolts L.E., representing the full introduction to the theory, and its second edition published in 1971 in collaboration with Norkin S.B. [22]). There were no studies with single point of view on numerous problems in the theory of functional-differential equations until the book by Hale J. (1977) [25].

Interpretation of solutions of functional-differential equations

$$\dot{x}(t) = f(x(t), t),$$

as integral curve in the space $R \times C$ by Krasovskii N.N. (1968) [32] served as such single point of view. This interpretation is now widespread, proved useful in many parts of the theory, particularly sections of the asymptotic behavior and periodicity of solutions. It clarified the functional structure of the functional-differential equations of delayed and neutral type, provided an opportunity to the deep analogy between the theory of such equations and the theory of ordinary differential equations and showed the reasons for deep differences of these theories.

Classic work on the theory of functional, integral and integro-differential equations is a work by Volterra V. [43].

The biggest part of the results obtained during 150 years before works by Volterra V. were related to special properties of very narrow classes of equations. In his studies of predator-prey models and studies on viscosity-elasticity Volterra V. got some fairly general differential equation, which include past states of system:

$$\dot{x}(t) = f(x(t), x(t - \tau), t), \quad \tau > 0.$$

In addition, because of the close connection between the equations and specific physical systems Volterra V. tried to introduce the concept of energy function for these models. Then he used the behavior of energy function to study the asymptotic behavior of the system in the distant future.

In late 1930 and early 1940s Minorsky N.F. in his article [36] very clearly pointed out the importance of considering the delay in feedback mechanism in his works on stabilizing the course of a ship and automatic control its movement.

At the beginning of 1950 Myshkis A.D. introduced general class of equations with delay arguments and laid the foundation for general theory of linear systems. In 1972 he systematized ideas in the paper [38]. Bellman R. showed in his monograph [6] a broad applicability of equations that contain information about the past in such fields as economics and biology. He also presented a well-constructed theory of linear equations with constant coefficients and the beginning of stability theory. The most intensive development of these ideas presented in the book of Bellman R. and Cooke K. [7]. The book describes the theory of linear differential-difference equations with constant and variable coefficients:

$$\dot{x}(t) = f(x(t), \dot{x}(t), ..., x^{(n)}(t), x(t-\tau_1), ...x(t-\tau_m), t),$$

$$\tau_i > 0, i = 1, ..., m.$$

Considerable attention is paid to asymptotic behavior of the solutions, as well as the stability theory of linear and quasi-linear equations. Most of the results in this area belong to these authors.

The book by Pinney E. [40] is devoted to differential- difference equations, otherwise known as the equations with deviating argument. The focus of the book is linear equations with constant coefficients, which are most often encountered in the theory of automatic control. The book also presents a new method for studying equations with small nonlinearities found by the author.

Azbelev N.V., Maksimov V.P., Rakhmatulina L.F. [3], Kurbatov V.G. [33] and Sabitov K.B. [41] are relatively new works to the theory.

1.1 Dynamical systems stability

One of the important characteristic of the dynamic system is stability of this system. The history of stability research is more than one century long and one of the first classical work in this branch of mathematic is book of Lyapunov A.M. [35]. This work contains author's results about stability of equilibrium and the motion of mechanical systems, the model theory for the stability of uniform turbulent liquid, and the study of particles under the influence of gravity. His work in the field of mathematical physics regarded the boundary value problem of the equation of Laplace. Lyapunov's method actually produced new branch for researching - Lyapunov stability problem.

In the book [2] Andronov A.A. and Pontryagin L.S. presented their results received from researching motion of dynamic system for which topologically trajectory doesn't change for small preturberation of the system. One of the main results of this work is well-known Andronov-Pontryagin criterion of topologically stability of dynamic system.

Krasovskii N.N. in his book on the theory of stability [29] introduced the theory of Lyapunov functionals, noting the important fact: some problems for such systems become more visual and easier to solve if the motion is considered in a functional space, even when the state variable is a finite-dimensional vector. The paper discusses some problems in the nonlinear systems of ordinary differential equations solutions stability theory. The justification of the Lyapunov functions method is adequately addressed, the existence of functions is clarified. Also the possibility of applying the method to study of the systems described by various ordinary differential equations apparatus is proved. He developed these methods further in his next works [30], [31].

Another method of stability research is frequency method. This method is developed in the works of Gelig A.H., Leonov G.A. [23].

1.2 Dynamical systems with delay

The future of many processes in the world around us depends not only on the present state, but is also significantly determined by the entire pre-history. Such systems occur in automatic control, economics, medicine, biology and other areas. Mathematical description of these processes can be done with the help of equations with delay, integral and integro-differential equations. Great contribution to the development of these directions is made by Bellman R., Lunel S.M.V., Mitropolskii U.A., Myshkis A.D., Norkin S.B., Hale J.C. [7], [26], [37], [38], [39].

Classical works in the field of differential equations with retarded argument are work by Myshkis A.D. [38] and Hale J.C. [25].

1.3 Dynamical systems of neutral type

There is also a large number of applications in which retarded argument is included not only as a state variable, but also in its derivative. This is so-called differential-difference equations of neutral type:

$$\dot{x}(t) = f(x(t), \dot{x}(t), \dot{x}(t-\tau)), \quad \tau > 0.$$

Problems that lead to such equations are more difficult to find, although they often appear in studies of two or more oscillatory systems with some links between them. Akhmerov R.R., Kamenskii M.I. and ot. [1], Bellman R., Cooke K. [7] and also Germanovich O.P. [24] raised questions regarding the systems of neutral type in their works.

1.4 Optimal dynamic systems control

The challenge of providing restrictions imposed on the movement of a dynamic system remains important task for theory and practice of management for a long time. The best-known approaches to solving this problem are based on the maximum principle and dynamic programming method of Bellman. Moreover, in these approaches, first of all, we seek the optimal control, which in addition to the optimality should also ensure some specified limits. However, the effective management of the system is not necessarily optimal, which allows to speak of a certain narrowness of these approaches. In this case, the procedure of synthesis is quite complex and is ineffective in high-dimensional system. Direct approaches to the synthesis of restrictions control on the system movement are also known.

Big research about practical problems of the theory of automatic control was presented by well-known scientist Lurie A.I. in (1951) [34].

One of fundamental works in control theory is the work by one of the primary researchers Kalman R.E. [27]. This work deals with further advances of the author's recent work on optimal design of control systems and Wiener filters. Specifically, the problem of designing a system to control a plant when not all state variables are measurable, or the measured state variables are contaminated with noise, and there are random disturbances is considered.

In full version of Ph.D. thesis proves of all following states are presented and illustrated with non-trivial examples.

2 Main definitions of the theory

2.1 Definitions of the control theory

Let Z be the state space of a dynamic system, U be the set of control functions, $z = z(z_0, u, t)$ be a vector characterizing the state of the dynamical system at the instant t, starting from the initial state z_0, $z_0 \in Z$, $z_0 = z(t_0)$ and the control function u, $u \in U$. Let X denote a subspace of Z and $x = x(z_0, u, t)$ be the projection of the state vector $z(z_0, u, t)$ onto X.

Definition 2.1 *The state z_0 is said to be controllable in the class U (controllable state), if there exist such control u, $u \in U$ and the number T, $t_0 \leq T < \infty$ that $z(z_0, u, T) = 0$.*

Definition 2.2 *The state z_0 is said to be controllable in the class U with respect to a given set X (relatively controllable state), if there exist such control $u \in U$ and the number T, $t_0 \leq T < \infty$ that $x(z_0, u, T) = 0$.*

Definition 2.3 *If every state z_0, $z_0 \in Z$ of a dynamic system is controllable, then we say that the system is controllable (controllable system).*

Definition 2.4 *If every state z_0, $z_0 \in X$ of a dynamic system is relatively controllable, then we say that the system is relatively controllable (relatively controllable system).*

Consider the following Cauchy's problem:

$$\begin{aligned} &\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) + Bu(t), \ \ t \in [0, T]\,, \ T < \infty, \\ &x(0) = x_0, \ \ x(t) = \varphi(t), \ \ -\tau \leq t < 0, \end{aligned} \tag{1}$$

where $x = (x_1(t), ..., x_n(t))^T$ is the phase coordinates vector, $x \in X$,
$u(t) = (u_1(t), ..., u_r(t))^T$ is the control function, $u \in U$, U is the set of piecewise-continuous functions,
A_0, A_1, B are constant matrices of dimensions $(n \times n)$, $(n \times n)$, $(n \times r)$ respectively, $\tau > 0$ is the constant delay.

The state space Z of this system is the set of n-dimensional functions

$$\{x(\theta), \ t - \tau \leq \theta \leq t\} \tag{2}$$

The space of the n-dimensional vectors x (phase space X) is a subspace of Z. The initial state z_0 of the system (1) is determined by conditions

$$z_0 = \{x_0(\theta), \ x_0(\theta) = \varphi(\theta), \ -\tau \leq \theta < 0, \ x(0) = x_0\}. \tag{3}$$

The state $z = z(z_0, u, t)$ of the system (1) in the space Z at the instant t is defined by trajectory segment (2) of phase space X.

Below we assume that the motions of system (1) take place for $t \geq 0$ in the space of continuous function. The initial function $\varphi(\theta)$ is taken to be piecewise-continuous.

In accordance with specified definitions state (3) we have defined, the system (1) is controllable if there exists such control u, $u \in U$ that $x(t) \equiv 0$, $T - \tau \leq t \leq T$ when $T < \infty$.

The state (3) of the system (1) is relatively controllable if there exists such control u, $u \in U$ that $x(T) = 0$ for $T < \infty$.

Remark 2.5 *The notion of a relatively controllable system follow from the specific nature of differential equations with delay. In the case of the usual differential equations ($A_1 = \Theta$), the sets Z and X coincide and, consequently, the notion of a "relatively controllable state" is equivalent to the well-known* [27] *term "controllable state".*

Let $\mathrm{X}_0(t)$ is a fundamental matrix of solutions of equation (1) in case when $B \equiv 0$, normalized in the point t_0, mean $\mathrm{X}_0(t_0) = I$. Let us define following function

$$\omega(t) = \mathrm{X}_0(t)B = \begin{pmatrix} \omega_1(t) \\ \ldots \\ \omega_n(t) \end{pmatrix},$$

where $\omega_i(t) = (\omega_{i1}(t), ..., \omega_{ir}(t))$, $i = 1, ..., n$.

Theorem 2.6 [27] *System* (1) *will be relatively controllable if and only if vector functions $\omega_i(t)$, $i = 1, ..., n$ are linearly independent on all time interval $t_0 \leq t \leq t_1$.*

Definition 2.7 *Delayed matrix exponential is a matrix function which has the form of a polynomial of degree k in intervals $(k-1)\tau \leq t \leq k\tau$, "glued" in knots $t = k\tau$, $k = 0, 1, 2, ...$:*

$$e_\tau^{At} = \begin{cases} \Theta, & -\infty < t < -\tau \\ I, & -\tau \leq t < 0 \\ I + A\frac{t}{1!} + A^2\frac{(t-\tau)^2}{2!} + ... + A^k\frac{(t-(k-1)\tau)^k}{k!}, & (k-1)\tau \leq t < k\tau, \\ & k = 1, 2, ... \end{cases}$$

where Θ is zero matrix.

Delayed matrix exponential was at first defined in [28] as fundamental matrix of solutions of the matrix differential equation with pure delay.

2.2 Definitions of the stability theory

Consider an autonomous nonlinear dynamical system

$$\dot{x}(t) = f(t, x(t)), \quad x(t_0) = x_0, \tag{4}$$

where $x(t) \in D \in R^n$ denotes the system state vector, D is an open set containing the origin, and $f : D \to R^n$ is continuous on D. Suppose (4) has a solution $\varphi(t)$.

Definition 2.8 *The solution $\varphi(t)$ of the system* (4) *is said to be Lyapunov's stable, if, for each $\varepsilon > 0$, there exists $\delta = \delta(e) > 0$ such that for every other solution $x(t)$ if $||x(t_0) - \varphi(t_0)|| < \delta$, then for each $t \geq 0$ holds*

$$||x(t) - \varphi(t)|| < \varepsilon,$$

where $|| \cdot ||$ is a norm.

Definition 2.9 *The solution $\varphi(t)$ of the system* (4) *is said to be asymptotically stable if it is Lyapunov's stable and if there exists $\delta > 0$ such that for every other solution $x(t)$ if $||x(t_0) - \varphi(t_0)|| < \delta$, then for each $t \geq 0$ holds*

$$\lim_{x \to \infty} ||x(t) - \varphi(t)|| = 0.$$

Definition 2.10 *The solution $\varphi(t)$ of the system* (4) *is said to be exponentially stable if it is asymptotically stable and if there exist positive constants α, β, δ such that for every other solution $x(t)$ if $||x(t_0) - \varphi(t_0)|| < \delta$, then for each $t \geq 0$ holds*

$$||x(t) - \varphi(t)|| \leq \alpha ||x(t_0) - \varphi(t_0)|| e^{-\beta t}.$$

Remark 2.11 *The stability investigation of an arbitrary solution $\varphi(t)$ can be easy reduced to the stability investigation of a zero solution $\dot{y}(t) \equiv 0$ using a simple substitute $x(t) = y(t) + \varphi(t)$, where $y(t)$ is a new unknown function.*

Definition 2.12 *Consider a functional $V(x) : R^n \to R$ such that:*

1. *$V(x) \geq 0$ with equality if and only if $x = 0$ (positive definite)*
2. *$\dot{V}(x) = \dfrac{dV(x)}{dt} \leq 0$ with equality if and only if $x = 0$ (negative definite).*

Then $V(x)$ is called a Lyapunov's functional.

Theorem (First Lyapunov's theorem)[35] *If there exists a positive definite Lyapunov's functional V(x) with a negative definite first derivative along the trajectories of a system of differential equations, then the solution of the system of differential equations is stable.*

Theorem (Second Lyapunov's theorem) [35] *If there exists a positive definite Lyapunov's functional V(x) such that for the derivation along the trajectories of a system of differential equations*

$$\dot{V}(x) = \frac{dV(x)}{dt} \leq W(x) < 0,$$

where $W(x)$ is some bounded function, then the solution of the system of differential equations is asymptotically stable.

3 Representation of the solution

3.1 Systems with same matrices

Let us consider the following Cauchy problem

$$\dot{x}(t) = Ax(t) + Ax(t-\tau) + f(t), \quad t \geq 0 \tag{5}$$

$$x(t) = \varphi(t), \quad -\tau \leq t \leq 0, \tag{6}$$

where $x(t) = (x_1(t), ..., x_n)^T$ is vector of states of the system,
$f(t) = (f_1(t), ..., f_n(t))^T$ is known function of disturbance, A is constant matrix of dimension $(n \times n)$, $\tau > 0, \tau \in R$ is a constant delay.

To solve Cauchy problem (5) - (6) let us find the fundamental matrix of solution of this equation. Fundamental matrix would be a solution of the following matrix equation

$$\dot{X}(t) = AX(t) + AX(t-\tau), \quad t \geq 0 \tag{7}$$

with initial condition

$$X(t) = I, \qquad -\tau \leq t \leq 0, \tag{8}$$

where $X(t)$ is matrix of type $(n \times n)$ and I is the identity matrix.

Theorem 3.1 [45] *The solution of equation* (7) *with identity initial condition* (8) *has the recurrent form:*

$$X_{k+1}(t) = \mathrm{e}^{A(t-k\tau)} X_k(k\tau) + \int_{k\tau}^{t} \mathrm{e}^{A(t-s)} A X_k(s-\tau) ds,$$

where $X_k(t)$ *is defined on the interval* $(k-1)\tau \leq t \leq k\tau$, $k = 0, 1, ...$

Theorem 3.2 [45] *The fundamental matrix of solutions of equation* (7) *has the form:*

$$\mathrm{X}_0 = \begin{cases} \Theta, & -\infty \leq t < -\tau \\ I, & -\tau \leq t < 0 \\ 2e^{At} - I, & 0 \leq t \leq \tau \\ 2e^{At} + 2e^{A(t-\tau)}\left(A(t-\tau) - I\right) + I, & \tau \leq t \leq 2\tau \\ \dots & \\ \sum\limits_{m=0}^{k-1} 2\mathrm{e}^{A(t-m\tau)} \sum\limits_{p=0}^{m} (-1)^{p+m} A^p \frac{(t-m\tau)^p}{p!} + (-I)^k, & (k-1)\tau \leq t < k\tau, \\ & k = 3, 4, ... \end{cases} \tag{9}$$

Theorem 3.3 [45] *The solution of homogeneous equation for equation* (5) *(mean* $f(t) \equiv 0$*) with initial condition* (6) *have the form:*

$$x(t) = \mathrm{X}_0(t)\varphi(-\tau) + \int_{-\tau}^{0} \mathrm{X}_0(t-\tau-s)\varphi'(s) ds,$$

where $\mathrm{X}_0(t)$ *is the fundamental solutions matrix* (9).

Theorem 3.4 [45] *The solution of the heterogeneous equation* (5) *with zero initial condition* $x(t) \equiv 0, -\tau < t < 0$, *has the form*

$$x(t) = \int_0^t \mathrm{X}_0(t - \tau - s) f(s) ds, \quad t \geq 0,$$

where $\mathrm{X}_0(t)$ *is the fundamental solutions matrix* (9).

Theorem 3.5 [45] *The solution of the heterogeneous equation* (5) *with the initial condition* (6) *has the form*

$$x(t) = \mathrm{X}_0(t)\varphi(-\tau) + \int_{-\tau}^0 \mathrm{X}_0(t - \tau - s)\varphi'(s) ds + \int_0^t \mathrm{X}_0(t - \tau - s) f(s) ds,$$

where $\mathrm{X}_0(t)$ *is the fundamental solutions matrix* (9).

3.2 Systems with commutative matrices

Let us consider the following Cauchy problem

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + f(t), \quad t \geq 0 \tag{10}$$

$$x(t) = \varphi(t), \quad -\tau \leq t \leq 0, \tag{11}$$

where $x(t) = (x_1(t), ..., x_n)^T$ is a vector of states of the system,
$f(t) = (f_1(t), ..., f_n(t))^T$ is known function of disturbance, A_0, A_1 are commutative constant matrices of dimensions $(n \times n)$, $\tau > 0$, $\tau \in R$ is a constant delay.

To solve Cauchy problem (10) - (11) let us find the fundamental matrix of solution of this equation. Fundamental matrix would be a solution of matrix equation

$$\dot{X}(t) = A_0 X(t) + A_1 X(t - \tau), \quad t \geq 0, \tag{12}$$

with initial condition

$$X(t) = I, \qquad -\tau \leq t \leq 0, \tag{13}$$

where $X(t)$ is matrix of type $(n \times n)$ and I is the identity matrix.
Now let us obtain the explicit form of the fundamental matrix of the system (12) for commutative matrices A_0, A_1.

Theorem 3.6 [45] *The solution of equation* (12) *with identity initial condition* (13) *has the recurrent form:*

$$X_{k+1}(t) = \mathrm{e}^{A_0(t - k\tau)} X_k(k\tau) + \int_{k\tau}^t \mathrm{e}^{A_0(t-s)} A_1 X_k(s - \tau) ds,$$

where $X_k(t)$ *is defined on the interval* $(k-1)\tau \leq t \leq k\tau$, $k = 0, 1, 2...$

Theorem 3.7 [49] *Let matrices A_0, A_1 of system* (12) *be commutative. Then the matrix*

$$X_0 = \begin{cases} \Theta, & -\infty \le t < -\tau \\ I, & -\tau \le t < 0 \\ e^{A_0 t}\left[I + Dt\right], & 0 \le t < \tau \\ \dots & \\ e^{A_0 t} e_{\tau}^{Dt}, & (k-1)\tau \le t < k\tau, \quad k = 1, 2, ..., \end{cases} \tag{14}$$

where $D = e^{-A_0\tau}A_1$, $t \ge 0$ is the solution of the system (12)*, satisfying the initial conditions* (13)*.*

Statements of the Theorems 3.3 - 3.5 are hold for the system with commutative matrices.

3.3 Systems with general matrices

Let us consider the following Cauchy problem

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) + f(t), \quad t \ge 0 \tag{15}$$

$$x(t) = \varphi(t), \quad -\tau \le t \le 0, \tag{16}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n)^T$ is a vector of states of the system,
$f(t) = (f_1(t), ..., f_n(t))^T$ is a known function of disturbance, A_0, A_1 are constant matrices of dimensions $(n \times n)$, $\tau > 0, \tau \in R$ is a constant delay.

To solve Cauchy problem (15) - (16) let us find the fundamental matrix of solution of this equation. Fundamental matrix would be a solution of the following matrix equation

$$\dot{X}(t) = A_0 X(t) + A_1 X(t-\tau), \quad t \ge 0, \tag{17}$$

with initial condition

$$X(t) = I, \qquad -\tau \le t \le 0, \tag{18}$$

where $X(t)$ is matrix of type $(n \times n)$ and I is the identity matrix.

Theorem 3.8 [44] *The solution of equation* (17) *with initial condition* (18) *has the recurrent form:*

$$X_{k+1}(t) = \mathrm{e}^{A_0(t-k\tau)} X_k(k\tau) + \int_{k\tau}^{t} \mathrm{e}^{A_0(t-s)} A_1 X_k(s-\tau) ds,$$

where $X_k(t)$ is defined on the interval $(k-1)\tau \le t \le k\tau$, $k = 0, 1, ...$

Theorem 3.9 *Fundamental matrix of solutions of equation* (17) *with identity initial conditions* (18) *has the following form:*

$$X_0 = \begin{cases} \Theta, & -\infty \le t < -\tau \\ I, & -\tau \le t < 0 \\ e^{A_0 t} + f_1(t), & 0 \le t \le \tau \\ e^{A_0 t} + e^{A_0(t-\tau)} f_1(\tau) + f_2(t), & \tau \le t \le 2\tau \\ \dots & \\ \sum\limits_{m=0}^{k-1} e^{A_0(t-m\tau)} f_m(m\tau) + f_k(t), & (k-1)\tau \le t < k, \\ & k = 3, 4, ... \end{cases} \tag{19}$$

where

$$f_p(t) = \sum_{\sum i_j = 1}^{p} \prod_{j=p}^{1} \left(\sum_{k_j=0}^{\infty} A_0^{k_j} A_1^{i_j} \right) \frac{(t-(p-1)\tau)^{K(p)}}{K(p)!} \prod_{s=p-1}^{1} \frac{\tau^{(1-i_{s+1})K(s)}}{(1-i_{s+1})K(s)!},$$

$$K(v) = k_v + i_v(1 + k_{v-1} + i_{v-1}(1 + ... + i_2(1 + k_1 + i_1)...)), \quad i_p = 1, i_j \in \{0, 1\}.$$

Statements of the Theorems 3.3 - 3.5 are hold for the system with general matrices.

4 Stability of the system with delay

4.1 Stability research

Let us consider the equation

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau), \quad t \geq 0, \tag{20}$$

with the initial condition

$$x(t) \equiv \varphi(t), \quad -\tau \leq t \leq 0,$$

where $x(t) = (x_1(t), ..., x_n)^T$ is a vector of states of the system, A_0, A_1 are constant matrices of dimensions $(n \times n)$, $\varphi(t)$ is vector of function, $\tau > 0$ is a constant delay.

In this section, we will investigate the stability of the delayed equation (20) with Lyapunov's second method. Let we construct the Lyapunov's functional in the form:

$$V(x) = x^T(t) H x(t),$$

where H is a symmetric, positive definite matrix.

Theorem 4.1 *If there exists a symmetric, positive definite matrix H such that*

$$\lambda_{min}(C) - 2|HA_1| \sqrt{\frac{\lambda_{max}(H)}{\lambda_{min}(H)}} > 0,$$

then the zero solution $y(t) \equiv 0$ of a system (20) is asymptotically stable for any $\tau > 0$.

Theorem 4.2 *Let system (20) is asymptotically stable, there we have the following evaluation of convergence of solution:*

$$||x(t)|| \leq - \left[\lambda_{min}(C) - 2|HA_1| \sqrt{\frac{\lambda_{max}(H)}{\lambda_{min}(H)}} \right]^{-1} \frac{dV(x(t))}{dt},$$

where $V(x(t)) = x^T(t) H x(t)$ is Lyapunov's functional.

5 Controllability of the system with delay

Let us have the control system of differential matrix equation

$$\dot{x}(t) = Ax(t) + Ax(t-\tau) + Bu(t), \quad t \geq 0, \tag{21}$$

with initial conditions $x(t) = \varphi(t)$, $-\tau \leq t \leq 0$ where $x = (x_1(t), ..., x_n(t))^T$ is a vector of states of the system, $u(t) = (u_1(t), ..., u_r(t))^T$ is a vector of control functions, A, B are constant matrices of dimensions $(n \times n)$, $(n \times r)$ respectively, $\tau > 0$ is a constant delay.

Theorem 5.1 *For relatively controllability of linear system with delay* (21) *is necessary and sufficient that rank*$(S) = n$, *where*

$$S = \{B \ AB \ A^2B \ ... \ A^{n-1}B\},$$

hence S is a matrix constructed by augmenting matrices B, AB,..., $A^{n-1}B$.

Let us consider the control system of differential matrix equation

$$\dot{x}(t) = A_0x(t) + A_1x(t-\tau) + Bu(t), \quad t \geq 0, \tag{22}$$

with initial conditions $x(t) = \varphi(t)$, $-\tau \leq t \leq 0$ where $x = (x_1(t), ..., x_n(t))^T$ is a vector of states of the system, $u(t) = (u_1(t), ..., u_r(t))^T$ is a vector of control functions, A_0, A_1 are commutative constant matrices of dimensions $(n \times n)$, B is constant matrix of dimension $(n \times r)$, $\tau > 0$ is a constant delay.

Theorem 5.2 [49] *For relatively controllability of the linear stationary system with delay* (22) *it is sufficient that for $(k-1)\tau \leq t \leq k\tau$ the rank$(S_k) = n$, where*

$$S_k = \{B \quad e^{-A_0\tau}A_1B \quad e^{-2A_0\tau}A_1^2B \quad ... \quad e^{-(k-1)A_0\tau}A_1^{k-1}B\},$$

hence S_k is a matrix constructed by augmenting matrices B, $e^{-A_0\tau}A_1B$, $e^{-2A_0\tau}A_1^2B$,..., $e^{-(k-1)A_0\tau}A_1^{k-1}B$.

Let us consider the control system of differential matrix equation

$$\dot{x}(t) = A_0x(t) + A_1x(t-\tau) + Bu(t), \quad t \geq 0, \tag{23}$$

with initial conditions $x(t) = \varphi(t)$, $-\tau \leq t \leq 0$ where $x = (x_1(t), ..., x_n(t))^T$ is a vector of states of the system, $u(t) = (u_1(t), ..., u_r(t))^T$ is a vector of control functions, A_0, A_1, B are constant matrices of dimensions $(n \times n)$, $(n \times n)$, $(n \times r)$ respectively, $\tau > 0$ is a constant delay.
Now we introduce for the equation (23) analogue of the characteristic equation

$$Q_i(s) = A_0Q_{i-1}(s) + A_1Q_{i-1}(s-\tau), \quad s \geq 0, i = 1, 2, ...$$

$$Q_0(0) = B, \quad Q_0(s) = \Theta, s \neq 0,$$

where Θ is zero matrix. Using the Hamilton-Kelly's formula, we notice that every matrix A_0^s, $s > n-1$ can be presented as linear combination of matrices A^j, $j = 0, ..., n-1$. Function takes for $0 \le s \le p\tau$ the following linear independent values:

	$s=0$	$s=\tau$	...	$s=p\tau$
$Q_0(s)$	B	Θ	...	Θ
$Q_1(s)$	A_0B	A_1B	...	Θ
$Q_2(s)$	A_0^2B	$(A_0A_1 + A_1A_0)B$	...	Θ
...	...	...	...	...
$Q_p(s)$	A_0^pB	$(A_0^{p-1}A_1 + ... + A_1A_0^{p-1})B$	...	A_1^pB
$Q_{p+1}(s)$	$A_0^{p+1}B$	$(A_0^pA_1 + ... + A_1A_0^p)B$	...	$(A_0A_1^p + ... + A_1^pA_0)B$
...	...	...	...	...
$Q_{n-1}(s)$	$A_0^{n-1}B$	$(A_0^{n-2}A_1 + ... + A_1A_0^{n-2})B$	...	$(A_0^{n-p-1}A_1^p + ... + A_1^pA_0^{n-p-1})B$
$Q_n(s)$	-	$(A_0^{n-1}A_1 + ... + A_1A_0^{n-1})B$	...	$(A_0^{n-p}A_1^p + ... + A_1^pA_0^{n-p})B$
...	...	...	...	...
$Q_{n+p-1}(s)$	-	-	...	$A_0^{n-1}A_1^pB$

Let us denote

$$Q = \{Q_0 \;\; Q_1 \;\; Q_2 \;\; ... \;\; Q_{n+p-1}\}$$
$$= \{Q_0(0) \;\; Q_1(0) \;\; Q_1(\tau) \;\; Q_2(0) \;\; Q_2(\tau) \;\; Q_2(2\tau) \;\; ... \;\; Q_{n+p-1}(p\tau)\}.$$

Theorem 5.3 *For relatively controllability of a linear stationary system with delay* (23) *it is sufficient that for* $(p-1)\tau \le t \le p\tau$ *will* $rank(Q) = n$, *where*

$$Q = \{B \;\; A_0B \;\; A_1B \;\; A_0^2B \;\; (A_0A_1 + A_1A_0)B \;\; A_1^2B \;\; A_0^3B$$

$$(A_0^2A_1 + A_0A_1A_0 + A_1A_0^2)B \;\; (A_0A_1^2 + A_1A_0A_1 + A_1^2A_0)B \;\; A_1^3B \;\; ... \;\; A_0^{n-1}A_1^pB\},$$

hence Q *is a matrix constructed by augmenting matrices* B, A_0B, A_1B, A_0^2B, $(A_0A_1 + A_1A_0)B$, A_1^2B, ... , $A_0^{n-1}A_1^pB$.

Remark 5.4 *Using the Hamilton-Kelly's formula, we notice that every matrix* A_1^s, $s > n-1$ *can be presented as linear combination of matrices* A_1^j, $j = 0, ..., n-1$, *so when* $k > n$ *for the system* (22) *we get* $S_k = S_n$, *and when* $p \ge n-1$ *for the system* (23) *we get for* $s \ge n\tau$ *linear dependent values, and matrix* Q *became*

$$Q = \{Q_0 \;\; Q_1 \;\; Q_2 \;\; ... \;\; Q_{2n-2}\}.$$

6 Control construction

Theorem 6.1 [51] *Let us have the control problem with delay with the same matrices* (21). *Let* $t_1 \geq (k-1)\tau$ *and the necessary and sufficient condition for controllability is implemented:*

$$rank(S) = rank\left(\{B\ AB\ A^2B\ ...\ A^{n-1}B\}\right) = n.$$

Then the control function can be taken as

$$u(\xi) = [\mathrm{X}_0(t_1-\tau-\xi)B]^T \left[\int_0^{t_1} \mathrm{X}_0(t_1-\tau-s)BB^T[\mathrm{X}_0(t_1-\tau-s)]^T ds\right]^{-1} \mu,$$

$$0 \leq \xi \leq t_1,$$

$$\text{where} \quad \mu = x_1 - \mathrm{X}_0(t_1)\varphi(-\tau) - \int_{-\tau}^{0} \mathrm{X}_0(t_1-\tau-s)\varphi'(s)ds,$$

and $\mathrm{X}_0(t)$ *is the fundamental matrix of solutions* (9) *on time interval* $t \geq (k-1)\tau$.

Theorem 6.2 [49] *Let we have the control problem with delay with the commutative matrices* (22). *Let* $t_1 \geq (k-1)\tau$ *and the sufficient conditions for controllability be implemented:*

$$rank(S_k) = rank\left\{B; e^{-A_0\tau}A_1B; e^{-2A_0\tau}A_1^2B; ...; e^{-(k-1)A_0\tau}A_1^{k-1}B\right\} = n,$$

Then the control function can be taken as

$$u(\xi) = [\mathrm{X}_0(t_1-\tau-\xi)B]^T \left[\int_0^{t_1} \mathrm{X}_0(t_1-\tau-s)BB^T\left[\mathrm{X}_0(t_1-\tau-s)\right]^T ds\right]^{-1} \mu,$$

$$0 \leq \xi \leq t_1,$$

$$\text{where} \quad \mu = x_1 - \mathrm{X}_0(t_1)\varphi(-\tau) - \int_{-\tau}^{0} \mathrm{X}_0(t_1-\tau-s)\varphi'(s)ds,$$

and $\mathrm{X}_0(t)$ *is the fundamental matrix of solutions* (14) *on time interval* $t \geq (k-1)\tau$.

Theorem 6.3 *Let us have the control problem with delay with general matrices* (23). *Let* $t_1 \geq (k-1)\tau$ *and the sufficient conditions for controllability be implemented:* $det(Q) = n$, *where the matrix* Q *was defined in Theorem 5.3. Then the control function can be taken as*

$$u(\xi) = [\mathrm{X}_0(t_1-\tau-\xi)B]^T \left[\int_0^{t_1} \mathrm{X}_0(t_1-\tau-s)BB^T\left[\mathrm{X}_0(t_1-\tau-s)\right]^T ds\right]^{-1} \mu,$$

$$0 \leq \xi \leq t_1,$$

$$\text{where} \quad \mu = x_1 - \mathrm{X}_0(t_1)\varphi(-\tau) - \int_{-\tau}^{0} \mathrm{X}_0(t_1-\tau-s)\varphi'(s)ds,$$

and $\mathrm{X}_0(t)$ *is the fundamental matrix of solutions* (19) *on time interval* $t \geq (k-1)\tau$.

7 Conclusions

In this thesis, a solution of the system of linear differential equation with delay in general form was built. There was presented the view of solutions for the system with same matrices, the system with commutative matrices and the general case matrices. Examples were given to illustrate the proposed solution.

The stability and the asymptotic stability of a solution of a certain class of a differential linear matrix equation with delay was investigated. The Lyapunovs functional has the basic role in the investigation. Example was given to illustrate the proposed method of investigation of the stability of the system.

Necessary and sufficient condition for controllability of differential linear matrix equation with the same matrices with delay was defined and the control was built. Sufficient conditions for controllability of differential equation with commutative matrices and general matrices with delay were also defined and the control was build. Examples were given also to illustrate the proposed controllability criterions and controls were build.

The prove of necessity of conditions from the Theorems 5.2 and 5.3 remains open problems.

Also open problem remains the construction the control function optimal due some criterion.

As future step to investigated can be consider the differential equation

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + w(t),$$

where $w(t)$ is a stochastic vector ("white noise").

Also it is open problem to construct controllability criterion for the system with non-constant delay

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - h(t)), \quad 0 < h(t) < t.$$

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Abstract

This work is devoted to computing the solution, stability of the solution and controllability of respective system of linear matrix differential equation with delay $\dot{x}(t) = A_0x(t) + A_1x(t-\tau)$, where A_0, A_1 are constant matrices and $\tau > 0$ is the constant delay. To solve this equation, the *step by step* method was used. The solution was found in recurrent form and in general form.

Stability and the asymptotic stability of the solution of the equation was investigated. Conditions for stability were defined. The Lyapunovs functional theory is basic for the investigation.

Necessary and sufficient condition for controllability in same matrices case was defined and the control was built. Sufficient conditions for controllability in communicative matrices case and general case were defined and controls were built.

Abstrakt

V předložené práci se zabýváme hledáním řešení lineární diferenciální maticové rovnice se zpožděním $\dot{x}(t) = A_0x(t) + A_1x(t-\tau)$, kde A_0, A_1 jsou konstantní matice, $\tau > 0$ je konstantní zpoždění. Dále se zabýváme odvozením podmínek stability řešení systému a řiditelnosti daného systému. Pro řešení tohoto systému byla použita metoda *krok za krokem*. Řešení bylo nalezeno jak v rekurentní formě tak i v obecném tvaru.

Je provedena analýza stability a asymptotické stability řešení systému. Jsou zformulovány podmínky stability. Hlavní roli v analýze stability měla metoda Ljapunovových funkcionálů.

Jsou zformulovány nutné a postačující podmínky řiditelnosti pro případ systémů se stejnými maticemi a je zkonstruována řídící funkce. Jsou odvozeny postačující podmínky pro řiditelnost v případě komutujících matic a v případě obecných matic a je sestrojena řídící funkce.