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RELIABILITY-BASED DESIGN AND ASSESSMENT OF STRUCTURES USING SURROGATE MODELS

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RELIABILITY-BASED DESIGN AND ASSESSMENT OF STRUCTURES USING SURROGATE MODELS

SPOLEHLIVOSTNÍ NÁVRH A POSOUZENÍ KONSTRUKCÍ S VYUŽITÍM NÁHRADNÍCH MODELŮ

THESIS OF A LECTURE FOR APPOINTMENT AS PROFESSOR
IN THE ELD OF STRUCTURAL AND TRANSPORT ENGINEERING

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
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1 Introduction

During my research activities, I have conducted methodological research in structural safety and reliability quantification, nonlinear fracture mechanics modeling and utilization of soft computing methods for solving inverse problems in materials and structural engineering. In this thesis, a part of this research focused on the use of soft computing methods in the reliability-based design and assessment of structures will be presented.

When developing methodologies and tools for inverse and forward reliability analyses, special emphasis was placed on the implementation of the most effective and powerful methods, models and procedures with respect to its primary focus on time-consuming tasks solved via nonlinear finite element method analysis. Examples of practical applications include determining the load-bearing capacity, reliability and remaining service life of existing concrete bridges (see the examples of the post-tensioned concrete bridge made of precast MPD-type girders in Fig. 1, top left, and the two-span reinforced concrete beam bridge in Fig. 1, top right) or reliability-based design optimization of mass-produced T– and TT–shaped structural elements (Fig. 1, bottom left and right).

In the thesis, methodological approaches for two types of reliability tasks will be pre-

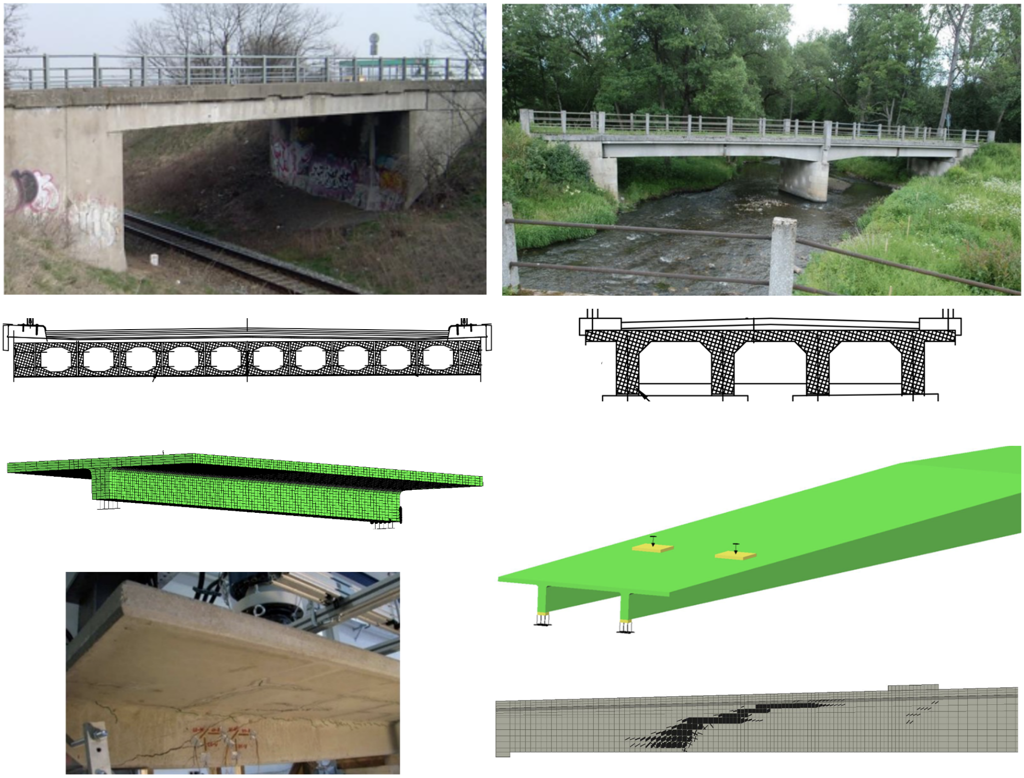

Figure 1: Examples of the analysed structures: inverse reliability analysis of the post-tensioned concrete bridge made of MPD girders (top left), the reinforced concrete beam bridge (top right), and reliability-based design optimization of T– and TT–shaped LDE 7 roof girders (bottom left and right).

sented: (i) reliability-based design of structures and (ii) metamodel-assisted reliability assessment of structures. In both cases, the uncertainties inherently present in the structure–load–environment system (e.g. material properties, geometrical imperfections, dead load, live load, wind, snow, corrosion rate, etc.) are taken into account using a fully probabilistic approach.

2 Reliability-based design of structures

To achieve desired level of reliability in limit state design is generally not an easy task. Traditional approaches simplified the problem by considering the uncertain parameters to be deterministic, and accounted for the uncertainties through the use of empirical safety factors. These factors are usually derived based on the past experience. But, they cannot guarantee required reliability level; they do not provide information on the influence of individual parameters on reliability. Also it is difficult (almost impossible) to design structures with uniform reliability levels among components.

Uncertainties in design variables and problem parameters are inevitable and must be considered in an optimization task, if reliable optimal solutions are to be found. For a canonical deterministic optimization task, the optimum solution usually lies on a constraint surface or at the intersection of more than one constraint surfaces (Figure 2). However, if the design variables or some system parameters cannot be achieved exactly and are uncertain with a known probability distribution of variation, the deterministic optimum (lying on one or more constraint surfaces) will fail to remain feasible in many occasions. In such scenarios, a stochastic optimization problem is usually formed and solved, in which the constraints are converted into probabilistic constraints meaning that probability of failures (of being a feasible solution) is limited to a pre-specified value.

When performing either reliability assessment or advanced engineering design, it is essential to take uncertainties into account using a probabilistic analysis. Reliability assessment requires forward reliability methods for estimating the reliability. On the other hand, the engineering design requires an inverse reliability approach to determine the design parameters to achieve desired target reliabilities.

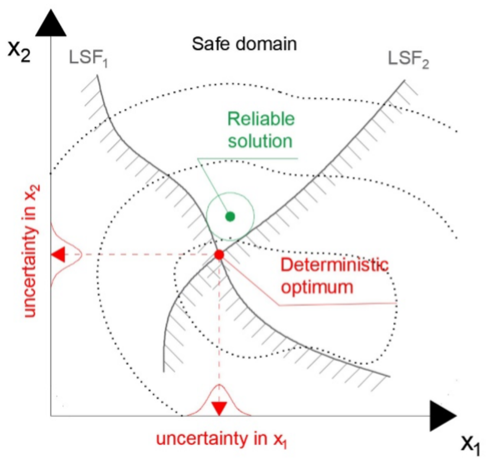

Figure 2: A comparison of deterministic and reliable solutions.

A "trial and error" procedure is generally used to determine the values of design parameters related to the design of particular limit states (both ultimate and serviceability, according to current standards). Design parameters (material properties, geometry, etc.) are changed step by step in order to satisfy specified limit states. This problem leads to the use of optimization methods. The task of achieving target reliability levels, expressed by theoretical failure probabilities or reliability indices, is much more difficult. The reliability problem is generally described by the limit state function and basic random variables. Design parameters can be deterministic or they can be associated with random variables described by statistical moments and a suitable model of probability distribution function (PDF). They affect the theoretical failure probability – a reliability indicator which cannot be easily calculated and requires an approximation method or the application of simulation techniques. The "trial and error" approach can also be used for the task of achieving target reliability levels, but its shortcomings are obvious.

While forward reliability methods have been applied widely and successfully in reliability engineering in various fields, inverse reliability approaches have not received the same degree of both attention and application, although they are particularly useful due to their important role in engineering design. The reason is that they are mathematically much more difficult – it is necessary to amalgamate forward reliability methods with other mathematical approaches of the optimization type. Inverse reliability problems appear when, for example:

1. Target reliability is specified in a design (ultimate and/or serviceability limit states). In this case, the design parameters must be determined to achieve the given reliability level.
2. Reliability-based design code is being calibrated. Code design procedures usually include performance and load safety factors, which are used to account for uncertainties and to produce a design with the desired reliability. To achieve this objective, the performance and load factors may be calculated using the inverse reliability approach.
3. A target quality is specified for a manufactured product. Several design parameters in the manufacturing process, ranging from material properties to process implementation, may have to be obtained in order to ensure that the processed product meets a pre-specified quality or tolerances with a desired reliability.

The methodology proposed by author attempts to overcome the shortcomings of existing inverse reliability methods. It utilizes artificial neural network (ANN) and a computational time is reduced by using a small-sample Latin hypercube sampling (LHS) simulation technique in ANN-based inverse problem proposed by Novák and Lehký [1], Lehký and Novák [2].

2.1 Reliability problem formulation

Classical reliability theory introduces the basic concept of structural reliability more formally, treating it as a response variable (e.g. deflection, stress, ultimate capacity, etc.) or safety margin Z (in the case that the function expresses a failure condition) which is the function of basic random variables $\mathbf{X} = X_1, X_2, \ldots, X_n$ (or random fields):

$$Z = g(X_1, X_2, ..., X_n), \tag{1}$$

where the function $g(\mathbf{X})$ represents a functional relationship between elements of vector $\mathbf{X}$ (computational model). Elements of vector $\mathbf{X}$ can be statistically correlated. If $g(\mathbf{X})$ represents a failure condition then it is called the limit state function or the performance function.

The structure is considered to be safe if:

$$Z = g(\mathbf{X}) = g(X_1, X_2, ..., X_n) > 0. \tag{2}$$

The limit state function can be an explicit or implicit function of basic random variables and it can take a simple or rather complicated form. Usually, the convention is that it takes a negative value if a failure event occurs; $Z \leq 0$, and the survival event is defined as $Z > 0$. The performance of the system and its components is described considering a number of limit states (multiple limit state functions). The aim of reliability analysis is the estimation of unreliability using a probability indicator called the theoretical failure probability, defined as:

$$p_\mathrm{f} = \mathrm{P}(Z \leq 0). \tag{3}$$

More formally, this probability is defined as:

$$p_\mathrm{f} = \int_{D_\mathrm{f}} f_\mathbf{X}(\mathbf{X})\,\mathrm{d}\mathbf{X}, \tag{4}$$

where the domain of integration is limited to the failure domain D_f where $g(\mathbf{X}) \leq 0$, $f_\mathbf{X}(\mathbf{X})$ is the joint probability density function of basic random variables (and also of other, deterministic quantities), and in general, its marginal variables can be statistically correlated. The explicit calculation of integral in Equation (4) is generally impossible. Therefore a large number of efficient stochastic analysis methods have been developed during the last decades. For practical calculations failure probability p_f can be substituted by the reliability index β, which makes the inverse reliability problem numerically more feasible to solve:

$$\beta = -\Phi^{-1}(p_\mathrm{f}), \tag{5}$$

where Φ is the distribution function of the standardised normal distribution. The variable safety margin in the original space of random variables together with the failure probability and reliability index is shown in Figure 3.

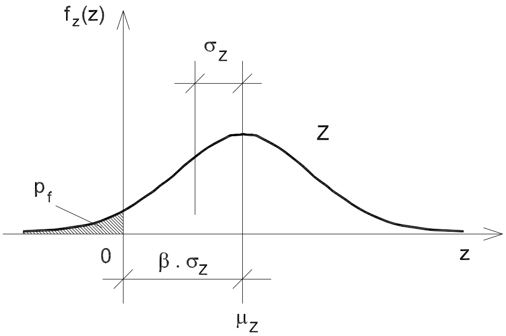

Figure 3: Safety margin, failure probability and Cornell's reliability index.

2.2 Inverse reliability method

The inverse reliability problem is the problem to find design parameters corresponding to specified reliability levels expressed by reliability index or by theoretical failure probability. In general, an inverse problem involves finding either a single design parameter to achieve a given single reliability constraint or multiple design parameters to meet specified multiple reliability constraints. The design parameters can be deterministic or they can be associated with random variables. Therefore, we include in addition to the vector of basic random variables $\mathbf{X} = X_1, X_2, \ldots, X_i, ..., X_n$ the vector of design deterministic parameters $\mathbf{d} = d_1, d_2, \ldots, d_k, ..., d_p$ and the vector of the design parameters of random variables $\mathbf{r} = r_1, r_2, \ldots, r_l, ..., r_q$. Note that the design parameters of random variables can be statistical moments of the first and/or second order. To consider higher statistical moments as design parameters is mathematically possible but useless from the practical point of view. In case of mean value one need to choose if either standard deviation or coefficient of variation will be fixed.

In the case of multiple limit states we have several safety margins Z_j and target failure probabilities $p_{\mathrm{f},j}$ or reliability indices β_j, where $j = 1, 2, ..., m$ is number of limit state functions. The inverse problem can be stated generally as:

$$\begin{aligned} \text{Given}: &\quad p_{\mathrm{f},j} \text{ or } \beta_j \\ \text{Find}: &\quad \mathbf{d} \text{ or/and } \mathbf{r} \\ \text{Subject to}: &\quad Z_j = g(\mathbf{X}, \mathbf{d}, \mathbf{r})_j = 0, \text{ for } j = 1, 2, ..., m. \end{aligned} \tag{6}$$

Table 1 shows alternatives which can occur for one variable (deterministic or random); design parameters to be found are marked by question mark.

Table 1: Design parameters alternatives.

Variable	Deterministic	Random	
		Mean	CoV
d_1	?	–	–
r_1	–	?	prescribed
r_2	–	prescribed	?
r_3, r_4	–	?	?

Note: d_1 is deterministic design parameter, $r_1 - r_4$ are design parameters associated with random variables.

A general soft computing-based inverse method is proposed and applied for solving inverse reliability problem, which aim is determination of the design parameters in order to achieve the prescribed reliability level. The inverse analysis is based on the coupling of a stratified LHS simulation method and an ANN. ANN as a cornerstone of the method is used as a surrogate model of unknown inverse function describing relation between the design parameters and corresponding reliability indicators.

$$\mathbf{P} = f_{\mathrm{ANN}}^{-1}(\mathbf{I}), \tag{7}$$

where $\mathbf{P} = \mathbf{d} \cup \mathbf{r}$ is the vector of all design parameters (determinist and random ones) and $\mathbf{I} = \boldsymbol{\beta} \vee \mathbf{p_f}$ is the vector of reliability indicators.

ANN has already been used for inverse reliability problems by some authors (Shayanfar et al. [3], Cheng et al. [4]). A novelty of the approach suggested here is the utilization of the efficient small-sample simulation method LHS [5, 6] used for the stochastic preparation of the training set utilized in training the ANN. For that purpose, the design parameters (e.g. mean values or standard deviations of selected random variables) are considered as random variables with a scatter reflecting the physical range of design values. Subsequently, the calculation of reliability is performed using appropriate simulation or approximation method, e.g. the First order reliability method (FORM). Once the ANN has been trained, it represents an approximation consequently utilized in a following way: To provide the best possible set of design parameters corresponding to prescribed reliability. The whole procedure is illustrated by a simple flow chart as shown in Figure 4 and is implemented as follows:

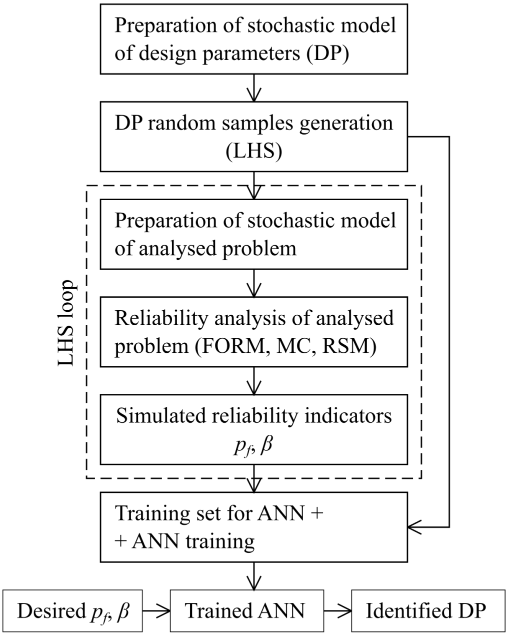

Figure 4: A flow chart of proposed inverse reliability method.

1. The limit state functions $g(\mathbf{X}, \mathbf{P})_j$ have to be defined first. This can be done at the level of explicitly defined formula or at the level of a computational model using the appropriate Finite element method (FEM) software. The functions have to be approximately calibrated via "trial and error" procedure using design parameters $\mathbf{P}$; the initial calculation uses a set of the initial design parameters resulting in a rough agreement with the target reliability indicators $\mathbf{I}$. An initial estimation of the design parameters has to be made based on engineering judgment and computational simulation. The parameters are estimated only roughly and therefore identification should follow as the next step.
2. Design parameters are considered as random variables described by a probability distribution; the rectangular distribution is a "natural choice" as the lower and upper limits represent the bounded range of the physical existence of design parameters. However, other distributions can also be used, e.g. Gaussian. Random realizations of design parameters are generated using LHS simulation method (see vector $\mathbf{P}$ in Figure 5).

3. A multiple calculation of reliability indicators related to the limit state functions using random realizations of design parameters is performed and a statistical set of the reliability indicators $\mathbf{I}$ is obtained (see Figure 5). Note that the selection of an appropriate number of simulations is driven by many factors, mainly by the complexity of the problem (computational demands), the structure of the neural network and the variability of design parameters. No general rule can be therefore suggested.
4. Random realizations $\mathbf{P}$ (outputs of ANN) and the random responses – reliability indicators related to the limit states $\mathbf{I}$ (inputs of ANN) – serve as the basis for the training of an appropriate ANN. This key point of the whole procedure is illustratively sketched in Figure 5 for $m = 2$ and $p + q = 2$ (see the definition of the inverse reliability problem at the beginning of this section).
5. The trained ANN is ready to provide an answer to the key task: To give the best design parameters so that the stochastic calculation may result in the best agreement with target reliability indicators, which is performed by means of a network simulation using target reliability indicators as an input. This results in an optimal set of design parameters $\mathbf{P}_{\text{opt}}$.
6. The last step is the verification of the results – the calculation of reliability indicators related to limit state functions using the optimal parameters $\mathbf{P}_{\text{opt}}$. A comparison with target reliability indicators will show the extent to which the inverse analysis was successful.

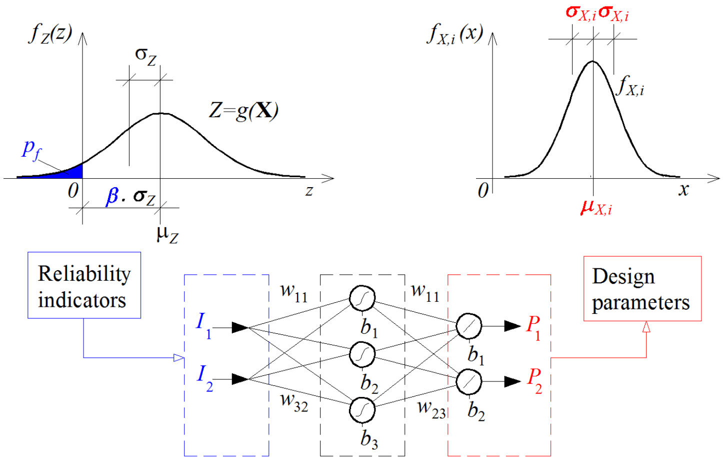

Figure 5: A schematic view of the artificial neural network-based inverse reliability method.

Note that the important step of the ANN-based inverse reliability method is the design of appropriate ANN structure (step 4), i.e. selection of the appropriate number of hidden layers and the corresponding number of neurons, the choice of transfer functions, etc. For more details about ANN theory and design see e.g. [7–9]. Let's just mention here that the number of inputs (reliability indicators corresponding to limit states) and the number of output neurons (design parameters) are known in advance.

In the case of inverse reliability analysis a double stochastic analysis is needed for the training set preparation for ANN (steps 2 and 3 of the procedure). In the outer loop random realizations of design parameters are generated using the LHS simulation technique. The inner loop represents the reliability calculation for one particular realization of design parameters. Here, the FORM or other approximation method [10] is recommended due to computational demands, as Monte Carlo (MC) type simulation techniques require a very high number of simulations for small failure probabilities (thousands, millions). The number of simulations in outer loop is driven by ANN and only tens of simulations are usually needed.

When solving the inverse reliability problem, finding the value of a design parameter can be difficult because it can have several solutions for the same design requirement. Figure 6 gives an illustration of such an issue. For the reliability index, there is an interval in feasible domain in which the inverse reliability task has more than one solution. In such a case, the original direct inverse reliability method may fail to locate all the solutions or even fail to achieve convergence if the initial design parameter is not chosen carefully. Two problems remain: 1) how to locate all solutions in the feasible domain, and 2) how to choose among multiple solutions. The only robust method is to consider all the feasible values of the design parameters, in small increments, repeating the forward reliability analysis, and choosing the solution corresponding to the target reliability level. Unfortunately, such a procedure is extremely computationally demanding. For such cases, a hybrid inverse reliability method (IRM) can be used combining the ANN-based inverse method with bisection method [11].

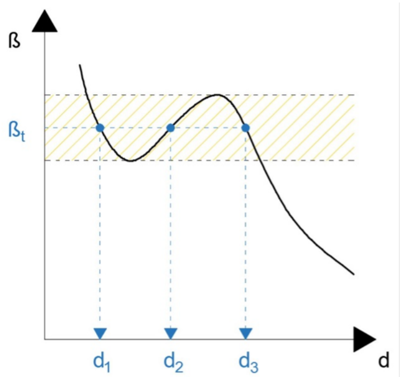

Figure 6: Example of relation between design parameter and reliability index.

The proposed hybrid method is able to locate solutions one by one as long as one can isolate intervals for them. This can be done using the equivalent deterministic analysis. Then, with initial design point $\mathbf{u}$ and initial design parameter d_0, the IRM is performed and an upgraded design parameter d is obtained. Once d is out of the given interval, instead of the IRM, the bisection method is used to obtain the upgraded d by linear interpolation or taking average of the bounds, which ensures the solution being in the interval. In the process of the iteration, the interval is narrowed using the newly obtained d. Conditional on the newly obtained d, the $\mathbf{u}$ vector is upgraded. The process is then repeated to seek the new d. The iterations repeat until the convergence is reached. The next solution can be obtained by providing another set of bounds and proceeding throughout the entire feasible domain. Selection of the most suitable solution can be then based on sensitivities to the reliability index. The advantage of this hybrid method is that it will never fail to find the solution with a reasonable convergence speed, since the bisection method will only be used when the solution is outside the bounds.

Since inverse reliability analysis combines ANN with multiple stochastic calculations, two software tools named DLNNET [12] and IRel [13] has been developed to automate such time-consuming tasks. DLNNET is the artificial neural network software which is combined with the FReET software for statistical, sensitivity and reliability analyses [14, 15]. Inverse reliability software IRel works as a master program which manages the whole process of inverse reliability analysis and controls communication between DLNNET and FReET. The ANN-based reliability method, together with the developed software, has been tested and applied on a number of theoretical and practical applications (e.g. the post-tensioned concrete bridge in Fig. 1 top left). For details, see [16–19].

2.3 Inverse reliability-based optimization

In order to find a unique solution to an inverse reliability problem with multiple design parameters, their number has to be equal to the total number of reliability constraints. In practice, however, the number of design parameters may be larger than the number of intervening constraints. In such case, one could find and infinite number of solutions satisfying the reliability constraints. A unique optimum solution could be obtained by introducing an optimization with an objective function related, for example, to structural cost. The problem thus become one of RBO problem which can be solved directly using small-sample double-loop RBO method.

As discussed above, in RBO, the reliability requirements are considered as nonlinear constraints in the optimization, requiring repeated forward reliability analysis. Since reliability calculation of complex structural systems is usually extremely time-consuming, an alternative approach called the inverse reliability-based optimization (IRBO) method was proposed. In contrast to traditional reliability-based optimization, this method permits the separation of the ordinary optimization method and the inverse reliability analysis [20]. The latter is performed to provide the equality constraints which can be eliminated by substituting them into the objective function. Any general optimization method can be used to obtain the independent (free) design parameters by performing optimization. In order to further reduce a computational effort the Aimed multilevel sampling (AMS) optimization method is employed [19] together with the small-sample simulation technique called Latin hypercube sampling [5].

The IRBO procedure illustrated by a simple flow chart in Figure 7 is implemented as follows:

1. According to number of reliability constraints, the free and dependent design parameters are determined.
2. Nonlinear optimization on the free design parameters is carried out. When requiring the evaluation of the objective function, an ANN-based inverse reliability analysis [16] is used to calculate dependent design parameters conditional on the upgraded free design parameters, thus the calculation of the objective function.
3. The iteration is repeated until convergence at the free design parameters is reached.

The simplest heuristic optimization method is to perform Monte Carlo type simulation within a design space and select the best random vector realization with regard to optimization criteria [21], [22]. Such a procedure clearly does not converge quickly toward a functional optimum and the quality of the solution depends on the number of simulations. The exact location of the optimum using only a simple simulation is highly improbable. The scatter of the results of such optimization is very high in the case of small sample analysis, and strongly dependent on the number of simulations. This approach however is very simple and requiring no knowledge of the features of the objective function. In addition, it is transparent, understandable and relatively easy to apply from the engineering point of view.

The AMS optimization method was first suggested in [23]. Its basic idea is to sort the

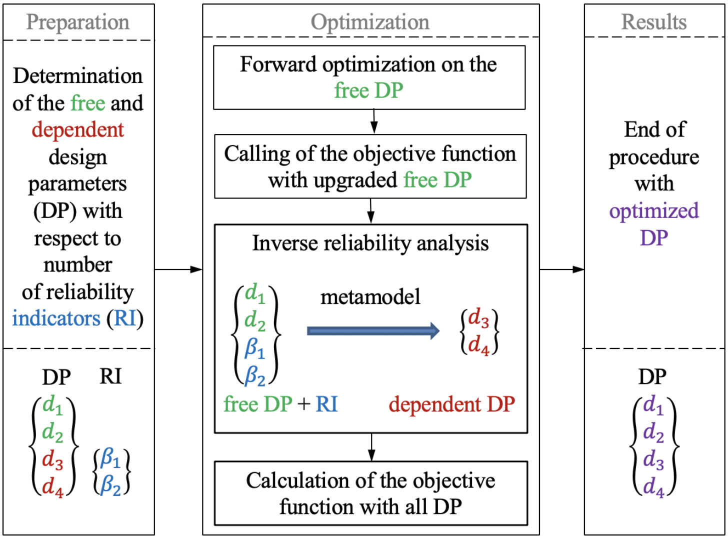

Figure 7: Flowchart of the inverse reliability-based optimization method.

course of the simulation into several levels. Advanced sampling within a defined space will be performed at each level. Subsequently, the sample with the best properties with regard to the definition of the optimization problem will be selected. Design vector $d_{i,\text{best}}$, which corresponds to the best realisation generated in the ith level, is determined as a vector of the mean values of the random variables selected for simulation within the next level of the AMS algorithm. Subsequently, the sampling space is scaled down around the best sample [24]. Another LHS simulation is then performed in this reduced space. This leads to a more detailed search in the area around the samples with the best properties with regard to the extreme value of the function. More detailed description of this method can be found in [19].

3 Metamodel-assisted reliability assessment of structures

According to current codes, structural design and load-bearing capacity assessment can be performed using both deterministic and probabilistic approaches. If the more advanced probabilistic approach is used, the reliability level related to a particular limit state is quantified via reliability indicators such as failure probability or reliability index, see Equations (3)–(5). In practice, nonlinear FEM simulation is a well-established approach to the analysis of structures. The response of a structure under serviceability as well as ultimate limit state conditions can be virtually simulated quite realistically. Nonlinear analysis using advanced material models of concrete and steel reinforcement which consider fracture properties, plasticity, relaxation, etc., enables the exploitation of reserves which are usually neglected or diminished in the codes or in linear analysis. From the failure probability calculation point of view, the utilization of FEM simulation means that the limit state function (LSF) $g(\mathbf{X})$ is not available as an explicit, closed-form function of the input variables. This reaffirms the need to use some of the available simulation or approximation methods.

The simulation methods are in general random numerical experiments which thus require a full analysis of the system for each generated set of loading and structural parameters. This means that a time-consuming structural analysis based on the FEM needs to be performed many times (ranging from several hundreds to millions of repetitions, depending on the expected failure probability value). This is the reason the utilization of the Monte Carlo method, a straightforward and well-established simulation method, is not feasible. As a partial solution, some variance reduction techniques can be employed, e.g., importance sampling [25], directional simulation [26], conditional expectation [27], and Latin hypercube sampling [5], which is often combined with the evaluation of Cornell's safety index, where the PDF of the safety margin is approximated by a normal distribution; see, e.g., [28]. Even with these improvements, the calculation is still quite time consuming.

Approximation methods (also known as metamodels, surrogate models, or response surface methods) seem to be the right alternative to simulation methods, which require enormous computational effort when calculating the failure probability of complex structural systems.

3.1 ANN-based response surface method

The general principle of the response surface method (RSM) is to replace the original LSF with an approximated (simpler) function whose evaluation is not so time consuming. The failure probability calculation is then performed via the utilization of classical simulation methods but with the approximated function instead of the original one. In original formulation of RSM, the LSF is approximated using a polynomial type function [29, 30]. Metamodels, which have gained popularity among researchers over the last few decades include polynomial chaos [31], support vector machine [32], and the Kriging [33, 34]. Nevertheless, with an increasing space of random variables, the number of discrete points (evaluations of the original LSF) needed to construct the response surface increase hand in hand with the time needed for reliability calculation. Therefore, in the case of large structures such as bridges, it is necessary to develop more efficient procedures that will reduce the number of evaluations of the original LSF to a minimum. Therefore, a small-sample artificial neural network-based response surface method (ANN-RSM) has been proposed. The ANN is used here as a surrogate model for the approximation of the original LSF. Thanks to its ability to generalize, ANN is more efficient at fitting the LSF even with a small number of simulations compared to the above-mentioned response surfaces. A stratified LHS method is utilized in the effective design of experiments in order to select the best neural network training set elements.

The small-sample ANN-RSM is based on the general methodology of inverse analysis [1,16]. The process of ANN-RSM application is divided into two main phases (see also Fig. 8): (1) the approximation phase, where the original LSF (in its explicit or implicit form) is approximated by a suitable ANN using a series of numerical experiments where the input random variables are selected according to the LHS simulation scheme and (2) the reliability calculation phase, where the approximated LSF is used instead of the original one in combination with classical simulation or approximation methods (e.g., Monte Carlo, FORM) for the calculation of reliability indicators (reliability index, failure probability).

The whole procedure is schematically illustrated using a simple flowchart in Figure 8, and can be itemized as follows:

1. Random realizations of input basic variables (possibly correlated) of an analyzed stochastic problem are sampled using the stratified LHS simulation method. The number of simulations depends on the complexity and required accuracy of the analyzed problem. In general, tens of simulations are used.
2. Evaluations of the original LSF are performed repeatedly for an individual vector of realizations of input random variables, and corresponding LSF values are calculated.

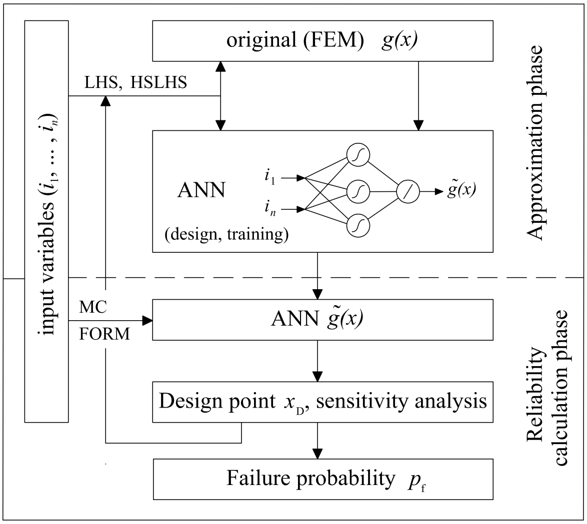

Figure 8: A flow chart of proposed inverse reliability method.

The FEM analysis can be used for the calculation of the original LSF.

3. The set of LSF values, together with a corresponding set of input random variables, serves as an ANN training set using an appropriate optimization technique (gradient descent methods, evolutionary methods, stochastic methods, etc.).
4. The trained ANN is subsequently used as a surrogate LSF for reliability analysis where reliability indicators are calculated via the utilization of classical simulation or approximation methods. In the case of simulation methods, millions of simulations can be used thanks to the extremely high speed of surrogate LSF evaluation compared to the original LSF.
5. In the case of poor convergence in the process of ANN training, or low accuracy, additional samples can be added using the hierarchical subset Latin hypercube sampling (HSLHS) strategy [35]. The approximated LSF can be further improved using a new anchor point near the design point according to:

$$\mathbf{X}_{\mathrm{M}} = \bar{\mathbf{X}} + (\mathbf{X}_{\mathrm{D}} - \bar{\mathbf{X}}) \frac{\tilde{g}(\bar{\mathbf{X}})}{\tilde{g}(\overline{\mathbf{X}}) - \tilde{g}(\mathbf{X}_{\mathrm{D}})}, \tag{8}$$

where the function $\tilde{g}(\mathbf{X})$ is used to obtain an estimate of the "design point", $\mathbf{X}_{\mathrm{D}}$, for the surface $\tilde{g}(\mathbf{X}) = 0$ based on the assumption of uncorrelated Gaussian variables. Once $\mathbf{X}_{\mathrm{D}}$ is found, $\tilde{g}(\mathbf{X}_{\mathrm{D}})$ is evaluated and a new anchor point, $\mathbf{X}_{\mathrm{M}}$, for interpolation is chosen on a straight line from the mean vector $\bar{\mathbf{X}}$ to $\mathbf{X}_{\mathrm{D}}$. This new anchor point in the multidimensional space of random variables is used as a set of updated mean values of such variables for new LHS sample generation, which is then located closer to the failure domain. In the case of standard deviations, we recommend reducing their size by half in each iteration to achieve even faster convergence.
6. The sensitivity analysis of the input random variables should also be performed before generating the additional samples. In the case of low dependence of the input random variable on the structural response, the variable can be omitted from the stochastic

model, which can in some cases significantly reduce the number of additional simulations.

For a detailed theory of the ANN-RSM method and its application to the assessment of several bridge structures, see Lehký and Šomodíková [10].

3.2 Inverse response surface method

As described in previous section, the response surface is an alternative to the real LSF. However, in contrast to the forward approach, when designing the structure, the function values that are used to construct the response surface are not available until the desired design variables are determined. Therefore, an inverse response surface method (IRSM) was proposed. It is based on a coupling of the adaptive RSM of Bucher and Bourgund [36] and the ANN-based inverse reliability method described in Section 2.2. The method is inspired by the procedure of Li [20], which combined the RSM with the Newton-Raphson iterative algorithm to solve inverse reliability problem [37]. The method proposed in this paper utilizes ANN and LHS methods which makes it more robust, efficient and therefore feasible for solving time-consuming problems such as structural design.

An iterative scheme to upgrade the response surface and, at the same time, to accomplish the inverse reliability analysis is proposed as follows:

1. In the first step of the IRSM, with the initial values for the design parameters, the initial response surface is constructed using foregoing RSM (polynomial, ANN, Kriging or any other metamodel can be employed). Based on this approximate response surface the ANN-based inverse reliability analysis is carried out and a new estimate of design parameters is obtained as well as the design point.
2. In the second step, the new anchor point is calculated from the design point using Equation (8). It serves together with the previously obtained design parameters for the response surface update. Based on this updated response surface the ANN-based inverse reliability analysis is carried out again to seek the new design parameters and the design point.
3. This process is repeated until the convergence is achieved at the design parameter with acceptable tolerance.

A graphical representation of the evolution of the response surface during the iterative process is given in the application section in Figure 10. For a detailed theory of the IRSM method its verification and practical application to the the post-tensioned concrete bridge mentioned in the Introduction (Figure 1 top left), see [38, 39].

4 Examples and applications

The presented methodologies for forward and inverse reliability analyses have been tested and applied to a wide range of theoretical and practical applications. For simplicity, illustration and due to the limited scope of the thesis, only two simple examples will be presented in this section, one on the IRBO of a simple steel beam and the other on the use of ISRM in solving a simple reliability problem. For application to more complex problems, I refer the reader to the following papers. In [16], the ANN-based inverse reliability method was tested and applied to various classes of problems – both linear and nonlinear cases with single as well as multiple design parameters, and with independent basic random variables as well as random variables with prescribed statistical correlations. In the paper [19], the reliability-based design optimization of a post-tensioned bridge structure made of MPD girders was addressed. The application of ANN-based RSM to various bridge structures is presented in [10]. The paper [39] details the application of adaptive IRSM method for prestressed concrete bridge.

4.1 Application of inverse reliability-based optimization

The first example represents a simple practical application of proposed IRBO procedure. Figure 9 shows simply supported beam made from thin-walled rectangular profile. Beam was subjected to a concentrated load in the middle. The span of beam was assumed 3.048 m, treated as a deterministic variable. Modulus of elasticity E, load P and cross-section dimensions t, B, H were taken as random variables. Their statistics are given in Table 2. The dimensions t and B were considered as the free design parameters, H was chosen as dependent design parameter. The goal is to find the dimensions of the cross-section t, B, H to ensure that the mid-span deflection exceeds 0.001016 m only with a given probabilities and corresponding reliability indices β_{target}. The goal is to minimize cross-sectional area and meet all constraint conditions as close as possible.

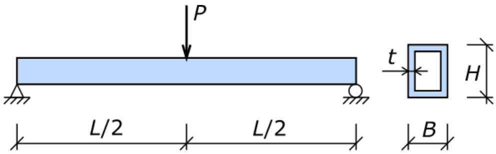

Figure 9: Scheme of the beam and cross-section.

Table 2: Stochastic model.

Variable	Distribution	Mean value	CoV
L (m)	Deterministic	3.048	–
E (GPa)	Normal	200	0.1
P (kN)	Normal	4.448	0.1
t (mm)	Normal	?	0.1
B (mm)	Normal	?	0.1
H (mm)	Normal	?	0.1

Optimization problem might be formulated as follows:
Find: the mean values of t, B and H, denoted by t_{m}, B_{m} and H_{m}.
Minimize: Cross-sectional area A_c

$$A_{\text{c}} = B_{\text{m}}H_{\text{m}} - (B_{\text{m}} - 2t_{\text{m}})(H_{\text{m}} - 2t_{\text{m}}). \tag{9}$$

Subject to reliability constraint $\beta = \beta_{\text{target}}$ for limit state function:

$$G = 1.016 - \frac{PL^3}{48EI}10^9, \tag{10}$$

where:

$$I = \frac{1}{12}BH^3 - \frac{1}{12}(B - 2t)(H - 2t) \tag{11}$$

and simple geometric constraints:

$$0.00254\,\mathrm{m} \leq t_\mathrm{m} \leq 0.0127\,\mathrm{m}, B_\mathrm{m} \geq 0.0127\,\mathrm{m}, H_\mathrm{m} \leq 0.381\,\mathrm{m}. \tag{12}$$

Reliability analysis was carried out using the FORM method; the starting values were means and the tolerance for convergence was 10^{-6}. Randomization of design parameters for training set preparation is displayed in Table 3. CoV stands for coefficient of variation, Std means standard deviation. One hundred random samples were generated using Latin hypercube sampling method. The ANN consisted of one hidden layer with five nonlinear neurons (hyperbolic tangent transfer function) and an output layer with one output neuron (linear transfer function) which corresponds to the dependent design parameter H. The ANN has three inputs which correspond to the two free design parameters t and B, plus one target reliability index β_target. Optimization was performed using AMS optimization method [23,24]. Geometric constraints were set according to Equation (12). The whole optimization process was divided to 10 levels with 300 simulations per level. A total of 3000 simulations were performed per optimization task.

Table 3: Randomization of design parameters for training set preparation.

Variable	Distribution	Mean value	Std	min	max
t_m (mm)	Rectangular	3.25	0.0433	2.5	4
B_m (mm)	Rectangular	210	51.962	120	300
H_m (mm)	Rectangular	325	43.301	250	400

The IRBO was carried out for different β_target and the optimum solutions are summarized in Table 4. The results show good accuracy of the obtained solution. At the same time, the activation of different geometrical conditions for different values of the required reliability is evident.

Table 4: Optimization results for different target reliability indices.

β_target (–)	t_m (mm)	B_m (mm)	H_m (mm)	A_c (mm^2)	β_t (–)
1	2.54	127	259.51	1937.67	0.997
3.09	2.54	127	350.40	2399.36	3.085
5	2.54	297.05	381	3418.70	5.001

4.2 Application of inverse response surface method

An explicit nonlinear LSF function adopted from [40] has been selected to show the IRSM procedure and demonstrate the validity and accuracy of the method. The complex solution of the function can be found in [41]. Here, the simplified form of the original function with only one standard normal variable X_1 and an identified deterministic parameter d (see the stochastic model in Table 5) was used in order to give the graphic interpretation of the proposed iteration scheme. An explicit function was defined as:

$$g(\mathbf{X}) = e^{[0.4(X_1+2)+6.2]} - e^{[d]} - 200. \tag{13}$$

The target reliability index is considered as $\beta_{\text{target}} = 2.688$, expected design parameter is $d = 5.163$ (calculated using 10 million Monte Carlo simulations of the original LSF). In order to construct the response surface and to perform ANN-based inverse reliability analysis the design parameter d has been treated as a uniformly distributed random variable. Two cases with the initial range, where the parameter d is inside this range (case 1) and out of the range (case 2), respectively, were used; see Table 6.

Table 5: Stochastic model.

Variable	Distribution	Mean value	Std
X_1	Normal	0	1
d	Deterministic	?	–

Table 6: Randomization of the design parameter.

Variable	Distribution	Mean value	Std	min	max
d – case 1	Rectangular	6	1.155	4	8
d – case 2	Rectangular	7	0.577	6	8

ANN-based response surface with two input neurons corresponding to variable X_1, and design parameter d, one linear output neuron corresponding to the value $g(\mathbf{X})$ and eight nonlinear neurons in hidden layer has been used to substitute the original LSF in Equation 13. The training set consists of 30 random samples of input parameter generated by LHS method. The gradient descent method with momentum was used for the ANN training.

Based on the constructed response surface the ANN-based inverse reliability analysis has been carried out. Utilized ANN consisted of two nonlinear neurons in a hidden layer and a linear output neuron corresponding to the design parameter d. There was one input to the network corresponding to reliability index. In order to create the training set, the reliability calculations using 1 million Monte Carlo simulations have been performed with 30 random samples of design parameter. After ANN training, the ANN was ready to provide the best design parameter related to the initial response surface. This was performed by means of a network simulation using target reliability index as an input.

With an updated design parameter, an updated response surface has been constructed for the next iteration. The stochastic model has been changed with respect to the updated design parameter and the new anchor point calculated according to Equation 8, i.e. random sampling was performed in a region closer to the design point. Standard deviation of the design parameter has been reduced to half of the original value in order to speed up the process and improve its convergence.

Table 7 shows the values of design parameter and reliability index during iteration process. In this case, the results reached the good accuracy after only two iterations. Let's note that reliability index β was calculated by 1 million Monte Carlo simulations of response surface. Figure 10 shows how response surface approaches the real LSF (g_{orig}) graphically. In the figure, there are two response surfaces, each depicted for both initial ("ini") value as well as updated ("upd") value of the design parameter in each iteration (1 and 2). Figure 10 also shows design points of all response surfaces (white bullets), i.e. the points that has the highest contribution to the probability integration.

Table 7: Results of IRSM iterative process.

Design parameters, reliability index	Identification – – iteration 1	Identification – – iteration 2	Target value
d – case 1	5.197	5.163	5.163
β – case 1	2.682	2.691	2.688
d – case 2	5.346	5.163	5.163
β – case 2	1.757	2.689	2.688

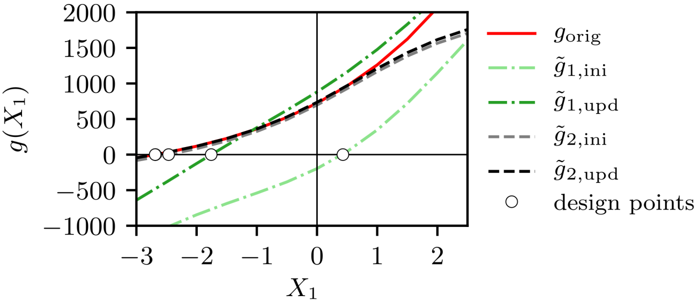

Figure 10: Evolution of response surfaces in iterative process for case 2.

From results, we can conclude that, even after couple of iterations, the iterative procedure significantly improves the quality of utilized response surface when performing structural reliability-based design. Generally, the initial ANN-based response surface approximation and consequent inverse reliability analysis cannot result in sufficiently accurate design parameter without RSM update. A number of iterations and successful convergence of the process is mostly dependent on the shape of the original limit state function and estimation of initial interval of the design parameter. As shown in case 2, even if the initial interval does not cover the actual value of the design parameter, the iterative procedure leads to correct results. This was also confirmed in other tested examples. However, the worse the estimation of the initial interval is, the worse the convergence of the iterative process can be. If the estimation is extremely poor then convergence may not be achieved.

Conclusion

Solving reliability-based design and probabilistic assessment of structural reliability are essential steps in a range of applications. Typical examples are the determination of reliability, load-bearing capacity and residual lifetime of aging bridges subjected to gradual deterioration which brings increasing level of uncertainty to its parameters. Reliability assessment and optimization are also an integral part of the development of mass-produced components or are used in the calibration of standards, etc. Proposed methods efficiently combines several techniques from stochastic mechanics and soft computing, which are usually utilized separately, in order to drastically reduce computational effort when performing reliability-based design and assessment of complex systems. In general, for complex systems, the response surface method is the only way to approach both forward analysis and inverse analysis since there is no other method which can give the solution with an acceptable level of computational effort.

When utilizing response surface method for inverse problems, the iterative procedure should be performed to ensure accuracy of reliability-based design and reliability assessment.

Reducing computation time is also extremely important for reliability-based optimization. When using the double-loop RBO method, performing the inner reliability loop is a computationally and time consuming part. Therefore, the proposed IRBO method permits the separation of the conventional optimization method and the inverse reliability analysis. It uses a surrogate model to explicitly approximate the inverse reliability with respect to design variables and reliability indicators. The outputs of the surrogate model are the values of the so-called dependent design variables related to the prescribed reliability constraints. Since these are equality constraints, no checks on the reliability constraints are required. Compared to the original double-loop RBO method, only a limited number of reliability analyses are required to construct the surrogate model. This leads to a drastic reduction in computational burden.

Future activities in research and teaching

Soft computing methods have become very popular in recent years, and in combination with traditional mathematical methods offer great potential to solving various types of engineering problems. Author plans to continue development of these methodologies with emphasis on their practical application, which requires a combination of several methods and software tools. Although the application of the presented methods will never be possible "in one click", for their practical use, user-friendly software tools need to be developed that are easy to use for users without deep knowledge of the whole methodology. Author already has experience in the development of similar software for the use of artificial neural networks in the identification of mechanical fracture parameters of quasi-brittle materials or inverse reliability analysis. Author's team is developing a computing environment based on the so-called node editor. The user links pre-programmed functional blocks (nodes) in the form of interfaces to individual programs and methods. This creates a visually clear flow chart (algorithm) of the solved task. Such a concept offers easy design of relatively complex procedures and also easy future extensibility with newly developed methods.

In terms of soft computing methods, the parallel use of multiple neural networks has great potential. Such a construct made up of many neural networks which are jointly used to solve a particular task is called a neural network ensemble (NNE). The fundamental mathematical idea of NNE rationally originates from "the weak law of large numbers in probability". From a practical point of view and with a view to minimizing computational effort, this theory will be combined with the mathematical optimization concept that "many could be better than all". It entails picking out excellent neural networks and eliminating the poorer ones via a specific procedure. The use of NNE can significantly improve the generalization capability and accuracy of the surrogate model while maintaining acceptable computational demands.

It is planned to further strengthen the research team with talented students from all levels of study and postdocs from cooperating foreign institutions. Priority will be given to applying for basic and applied research projects and cooperation with industrial partners on the implementation of theoretical knowledge into practice. The international prestige of the team will be strengthened through continued international collaboration, making international internships available to students and researchers, teaching and invited lectures at foreign institutions, and serving on international committees and organizations.

Within the teaching activities, the author wants to continue to incorporate the latest research findings into teaching at all levels of study, and thus contribute to improving the quality of teaching and preparing highly qualified graduates. It s also planned to continue individual work with students, supervising bachelor's, master's and dissertation theses. Talented students will be offered the opportunity to participate in research activities and support their mobility to foreign universities and conferences. Such motivated students may continue their scientific career at the university in the future or find employment in industry with the potential to collaborate on applied research projects.

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Shrnutí

Předložená práce představuje část výzkumné činnosti autora zaměřené na využití metod soft computing při spolehlivostním návrhu a posouzení stavebních konstrukcí. Při vývoji metodik a nástrojů pro inverzní a dopřednou analýzu spolehlivosti byl kladen zvláštní důraz na implementaci co nejefektivnějších a nejvýkonnějších metod, modelů a postupů s ohledem na jejich primární zaměření na časově náročné úlohy řešené pomocí nelineární analýzy metodou konečných prvků. V práci jsou postupně představeny metodické postupy pro dva typy úloh spolehlivosti: (i) spolehlivostní návrh konstrukcí a (ii) posouzení spolehlivosti konstrukcí s využitím metamodelování. V obou případech jsou zohledněny vstupní nejistoty, které jsou přirozeně přítomny v celém systému konstrukce–zatížení–prostředí (např. vlastnosti materiálu, geometrické imperfekce, stálá zatížení, nahodilá zatížení, vítr, sníh, míra koroze atd.), a to s využitím plně pravděpodobnostního přístupu.

Spolehlivostní návrh a optimalizace konstrukcí patří mezi tzv. inverzní úlohy, kdy pro analyzovaný mezní stav a jemu odpovídající požadovanou úroveň spolehlivosti definovanou tzv. ukazateli spolehlivosti (pravděpodobnost poruchy či index spolehlivosti) hledáme hodnoty tzv. návrhových parametrů. Ty mohou být deterministické nebo se může jednat o statistické charakteristiky náhodných veličin. Předmětem návrhu často bývá sada návrhových parametrů a je požadováno splnění požadované spolehlivosti pro více mezních stavů. Běžnou praxí při deterministicky definované úloze je použití metody pokus–omyl. Ta je však pro řešení výše definované inverzní úlohy spolehlivosti nepoužitelná. Nabízí se využití přímé spolehlivostní dvousmyčkové optimalizace, která však může být v případě konstrukcí řešených nelineární metodou konečných prvků poměrně časově náročná. Práce proto představuje metodu inverzní analýzy spolehlivosti založenou na umělé neuronové síti a stratifikované simulační metodě Latin hypercube sampling. Základním stavebním kamenem metody je umělá neuronová síť, která slouží jako náhradní model neznámé inverzní funkce popisující vztah mezi návrhovými parametry a odpovídajícími ukazateli spolehlivosti. Stratifikovaná simulační metoda pak slouží pro efektivní návrh tzv. učící množiny nezbytné pro nastavení parametrů sítě.

Aby bylo možné nalézt jedinečné řešení inverzního problému spolehlivosti s více návrhovými parametry, musí být jejich počet roven celkovému počtu spolehlivostních okrajových podmínek. V praxi však může být počet návrhových parametrů větší než počet limitujících ukazatelů spolehlivosti. V takovém případě existuje nekonečný počet řešení vyhovujících daným spolehlivostním podmínkám. Jedinečné optimální řešení by bylo možné získat zavedením optimalizace s cílovou funkcí vztahující se například k ceně konstrukce či její opravy. Z úlohy se tak stává problém spolehlivostní optimalizace, který lze řešit výše uvedenou dvousmyčkovou metodou. Ta umožňuje oddělit optimalizační část (vnější smyčka) a část spolehlivostní (vnitřní smyčka). Pro co nejefektivnější vyřešení úlohy se dále nabízí nahradit vnitřní spolehlivostní smyčku výše uvedenou metodou inverzní spolehlivosti založenou na umělé neuronové síti. Ve srovnání s klasickou dvojmyčkovou metodou je pak zapotřebí provést pouze omezený počet spolehlivostních analýz, což vede k výraznému snížení výpočetní náročnosti.

Jak je uvedeno výše, úroveň spolehlivosti vztahující se k určitému meznímu stavu je kvantifikována pomocí ukazatelů spolehlivosti, jako je pravděpodobnost poruchy nebo index spolehlivosti. V praxi se k numerickým analýzám konstrukcí využívá lineární či nelineární simulace metodou konečných prvků (MKP). Z hlediska výpočtu pravděpodobnosti poruchy znamená využití MKP absenci funkce poruchy v explicitním uzavřeném tvaru, a tedy nutnost použít některou z dostupných simulačních nebo aproximačních metod pro stanovení ukazatelů spolehlivosti. Druhá část práce proto pojednává o využití metamodelování při stanovení spolehlivosti konstrukcí. V práci je představena aproximační metoda plochy odezvy (response surface method) založená na umělé neuronové síti, která slouží jako náhradní model původní funkce poruchy. Ve výchozí konfiguraci je funkce poruchy nahrazena v celém oboru hodnot, následně je provedeno zpřesnění aproximace v oblasti poruchy přesunem k tzv. návrhovému bodu. Jelikož vyčíslení náhradní funkce je v porovnání s původní funkcí poruchy o několik řádů rychlejší, výpočet ukazatelů spolehlivosti pak může proběhnout s pomocí vhodné simulační metody, např. Monte Carlo.

Poslední představenou metodou je metoda inverzní plochy odezvy (inverse response surface method). Jak vyplývá z názvu, svoje uplatnění najde při využití náhradního modelu v rámci spolehlivostního návrhu. Na rozdíl od přímé analýzy spolehlivosti totiž nejsou při návrhu konstrukce známy hodnoty funkce spolehlivosti, které se používají ke konstrukci plochy odezvy (náhradního modelu), a to proto, že nejsou známy hodnoty návrhových parametrů. Pro tyto účely byla navržena metoda inverzní plochy odezvy, kdy dochází k postupnému adaptování plochy odezvy ruku v ruce s postupným zpřesňováním hodnot návrhových parametrů za pomocí inverzní analýzy spolehlivosti představené v první části práce.

V poslední části práce jsou pro názornost ukázány na dvou vybraných jednoduchých aplikacích postupy a dílčí výsledky spolehlivostní optimalizace a metody inverzní plochy odezvy. Pokročilejší a praktičtější aplikace na reálné konstrukce je pak možné najít v odkazovaných publikacích autora.