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**TRANSMISSION LOSS ALLOCATION**  
**ALOKACE ZTRÁT V PŘENOSOVÉ SOUSTAVĚ**

ZKRÁCENÁ VERZE PH.D. THESIS

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# 1 INTRODUCTION

## 1.1 PROBLEM FORMULATION

The electric power industry all over the world has been undergoing a principal structural change that consists in unbundling the generation, transmission and distribution of electricity. This means that once vertically integrated companies covering the whole path from generation to customers are split into independent parts taking care either about production, transmission or distribution. The aim of this process is introducing competition on the side of generation and distribution, which should result in increasing the efficiency of the electricity sector and, consequently, in decreasing the price of electricity for customers.

The opening of the power industry to competitive forces – known as the liberalization or deregulation of electricity markets – must, however, respect the fact that the transmission system represents a natural monopoly. In other words, from the economic point of view it is not possible to create another competitive transmission system. The transmission system thus plays a special role in deregulated electricity sectors because, “in order to ensure transparency and non-discrimination, the transmission function of vertically integrated undertakings should be operated independently from the other activities” [60]. This is assured by establishing an independent transmission system operator that takes care of the operation, maintenance and, if need be, expansion of the system.

One of the important issues connected with the operation of the transmission system is the fact that the users of the system, i.e. generators and loads, influence power flows in the system and are thus responsible for the transmission losses. Since the losses are nonlinear functions of power flows in the lines of the system, it is impossible to associate directly a given user (generator or load) with a corresponding part of the transmission losses. Still, the transmission losses must be covered by the users and, consequently, a procedure that allows assigning the responsibility of users for a part of the losses must be found. Such a procedure is called transmission loss allocation.

As we have seen, an absolutely fair allocation method cannot exist, but an allocation algorithm should respect the value of bus power as well as the relative position of buses in the system (loads close to generators contribute less to losses than more distant loads and this should be reflected). Also, losses should be allocated both to generators and loads because both of them contribute to power flows in the system and this fact should result in balanced allocation of losses between generators and loads.

## 1.2 OVERVIEW OF EXISTING APPROACHES

Transmission loss allocation has been widely discussed in the literature, especially over the past five years. The proposed methods of transmission loss allocation can be divided into three groups

- **Pro Rata Allocation**, where losses are allocated proportionally to the real power of generation or load. Usually 50 % of the losses are allocated to generators and 50 % to loads, but another distribution between generation and load can also be found (e.g. in Spain 100 % of losses are allocated to loads). The advantage of this method is a very simple algorithm, on the other hand the fact that it completely ignores the relative position of buses (i.e. users) in the system is considered to be its weak point. See [11], [53].
- **Proportional Sharing Allocation**, where the allocation is based on the idea that a contribution of each generator and load to power flow in each line can be traced. It offers quite a simple algorithm and in a certain way takes into account the relative position of buses. The question is whether the method represents really the situation in the system. See [4], [11], [53].
- **Incremental Transmission Loss Coefficient Allocation**, where losses are allocated to generators and loads by means of incremental transmission loss coefficients. It is believed to be the method that represents in the most accurate way physical phenomena in the system. The principal disadvantage is a complicated solution of incremental loss coefficients that leads to various simplifications affecting – sometimes quite dramatically – the precision of the solution. Moreover, in this method one of the buses must be chosen as the slack bus with no losses allocated (incremental loss coefficients of the slack bus are equal to zero). It should be also noted that the existing methods neglect the influence of the bus reactive power on the system losses. See [11], [13], [14], [53].

Let us emphasize here that none of allocation methods has been fully accepted as the ideal one. As the practical implementation is concerned, mostly variations of pro rata allocation method are used, mainly because of its simplicity. The only practical application of incremental allocation up to now represents the Norwegian power system.

### **1.3 GOALS OF THE DISSERTATION**

The primary goal is to propose a new allocation method (or methods) using incremental transmission loss coefficients. The last of the possible approaches – incremental transmission loss coefficient allocation – has been chosen for two reasons. Firstly, as it has been already stated, it is believed to be the most faithful method from the point of view of physical phenomena in the system. Secondly, while the first two approaches can be taken for theoretically solved problems, incremental transmission loss coefficient allocation still represents – due to the complicated solution of incremental loss coefficients – a challenging area for research activities. The proposed incremental method(s) together with the other two known approaches (pro rata and proportional sharing allocation) will be tested on sample systems; the results will be analyzed in order to compare the used methods.

A secondary goal of the dissertation is to introduce voltage-dependent loads in load flow solution by a modification of the Newton-Raphson algorithm, which should lead to higher precision of the algorithm.

## **2 LOAD FLOW SOLUTION**

Load flow solution consists of the calculations of the bus voltages in a given power system that can be then used for calculating power flows and losses. Load flow solution thus provides key data necessary for tasks concerning various aspects of the operation of power systems, such as power system control, power system expansion analysis or – as it is the case in this work – allocation of transmission losses.

Theoretically, load flow solution can be represented as a linear problem. In other words, loads and supplies at buses of the system are given in the form of currents. In reality, the real and reactive power of loads and supplies are given, which means that the problem is expressed by a set of nonlinear equations. The solution then consists in the application of an iterative method. Two methods are mostly used in power engineering: the Gauss-Seidel method and the Newton-Raphson method.

The Gauss-Seidel method offers a very simple algorithm. On the other hand, it converges very slowly and it is necessary to use very high precision of the process in order to obtain solution (bus voltages) in sufficient precision. That is why the Newton-Raphson iterative method is usually applied to load flow solution.

Generally, it is supposed that the real and reactive power of loads are constants that do not change either with voltage or with time. In fact, there is a relationship between power and voltage as well as a time dependence of load. These dependences can be represented by a load model. A static model describes a relationship between power and voltage while a dynamic model describes a relationship between power and time. There can also be a model that combines both types of relationship. It is obvious that in case of transmission loss allocation (that is based on results of load solution) we can talk about a steady state of the power system and thus only the change of power with voltage can be taken into account.

Thus an appropriate static load model that represents the relationship between power and voltage by means of voltage-dependent load characteristics should be found. Let us choose an exponential model

$$\begin{aligned}\frac{P_i}{P_{in}} &= \left(\frac{V_i}{V_n}\right)^{x_i} \\ \frac{Q_i}{Q_{in}} &= \left(\frac{V_i}{V_n}\right)^{y_i}\end{aligned}\tag{2.1}$$

where  $P_i$  real power at bus  $i$  at voltage  $V_i$ ,  
 $Q_i$  reactive power at bus  $i$  at voltage  $V_i$ ,  
 $P_{in}$  real power at bus  $i$  at nominal voltage  $V_n$ ,  
 $Q_{in}$  reactive power at bus  $i$  at nominal voltage  $V_n$ ,

and incorporate it to load flow solution. This modification was published in [8]. Load flow equations can be expressed as

$$\begin{aligned}P_i &= \sum_{j=1}^n V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ Q_i &= \sum_{j=1}^n V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij})\end{aligned}\tag{2.2}$$

where  $V_i, V_j$  bus voltages magnitudes,  
 $\delta_{ij} = \delta_i - \delta_j$  difference of bus voltages phase angles,  
 $G_{ij}$  real part of element  $\bar{Y}_{ij}$  of the bus admittance matrix  $[\bar{Y}]$ ,  
 $B_{ij}$  imaginary part of element  $\bar{Y}_{ij}$  of the bus admittance matrix  $[\bar{Y}]$ .

The modified algorithm uses the following equations

$$\begin{aligned}P_{in} &= V_n^{x_i} \sum_{j=1}^n V_i^{1-x_i} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ Q_{in} &= V_n^{y_i} \sum_{j=1}^n V_i^{1-y_i} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij})\end{aligned}\tag{2.3}$$

and it can be written in matrix form as



$$\begin{bmatrix} [\Delta P_n] \\ [\Delta Q_n] \end{bmatrix} = \begin{bmatrix} \left[ \frac{\partial P_n}{\partial \delta} \right] & \left[ \frac{\partial P_n}{\partial V} \right] \\ \left[ \frac{\partial Q_n}{\partial \delta} \right] & \left[ \frac{\partial Q_n}{\partial V} \right] \end{bmatrix} \begin{bmatrix} [\Delta \delta] \\ [\Delta V] \end{bmatrix} \quad (2.4)$$

or

$$\begin{bmatrix} [\Delta P_n] \\ [\Delta Q_n] \end{bmatrix} = \begin{bmatrix} [C] & [D] \\ [E] & [F] \end{bmatrix} \begin{bmatrix} [\Delta \delta] \\ [\Delta V] \end{bmatrix} \quad (2.5)$$

The elements of the Jacobian are derived from Equations (2.3).

$$C_{ii} = -V_n^{x_i} V_i^{2-x_i} B_{ii} - V_n^{x_i-y_i} V_i^{y_i-x_i} Q_{in} \quad (2.6)$$

$$C_{ij} = V_n^{x_i} V_i^{1-x_i} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \quad (2.7)$$

$$D_{ii} = V_n^{x_i} V_i^{1-x_i} G_{ii} + \frac{1-x_i}{V_i} P_{in} \quad (2.8)$$

$$D_{ij} = V_n^{x_i} V_i^{1-x_i} (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \quad (2.9)$$

$$E_{ii} = -V_n^{y_i} V_i^{2-y_i} G_{ii} + V_n^{y_i-x_i} V_i^{x_i-y_i} P_{in} \quad (2.10)$$

$$E_{ij} = -V_n^{y_i} V_i^{1-y_i} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \quad (2.11)$$

$$F_{ii} = -V_n^{y_i} V_i^{1-y_i} B_{ii} + \frac{1-y_i}{V_i} Q_{in} \quad (2.12)$$

$$F_{ij} = V_n^{y_i} V_i^{1-y_i} (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \quad (2.13)$$

The iterative process is analogical to the standard Newton-Raphson method. It should be noted, however, that during the solution the real and reactive power at *nominal* voltage is used, which means that the iterative process is repeated until the following is true

$$\begin{aligned} |\Delta P_{in}^{(r)}| &\leq \varepsilon \\ |\Delta Q_{in}^{(r)}| &\leq \varepsilon \end{aligned} \tag{2.14}$$

where  $r$                       number of iteration,  
 $\varepsilon$                               required precision of the process  
 $\Delta P_{in}^{(r)}$                       real power difference at nominal voltage for bus  $i$  at iteration step  $r$ ,  
 $\Delta Q_{in}^{(r)}$                       reactive power difference at nominal voltage for bus  $i$  at iteration step  $r$ .

### 3 ALLOCATION OF TRANSMISSION LOSSES

Generally, losses represent the difference between supplied and consumed power. From the point of view of the entire power system, losses can be divided into two principal categories: technical and non-technical losses. Non-technical losses are mainly caused by improper metering and unauthorized loads. Technical losses represent resistive ( $I^2R$ ) losses, losses due conductance (current leakage and corona) as well as transformation losses. Technical losses in the power system can amount to 10 percent of the supply.

This dissertation, however, deals with the transmission losses that represent about 1-2 percent of the supply, which corresponds to tens or even hundreds of megawatts according to the size of the system.

As it has been already stated, the purpose of transmission loss allocation is the assigning of a corresponding part of incurred losses in the transmission system to its individual users. Three basic approaches have been reported in the literature

- *pro rata allocation*, with the algorithm based on the value of the bus real power,
- *proportional sharing allocation*, where it is supposed that a contribution of each generator and load to power flow in each line can be traced,
- *incremental allocation*, with the algorithm using incremental transmission loss coefficients.

The author would like to reiterate that since losses are nonlinear functions of line flows and – consequently – bus powers, it is not possible to find a perfectly fair allocation method that could not be questioned. All of the existing methods involve a certain level arbitrariness.

The allocation of losses thus remains an open issue. Still, we can define requirements that a loss allocation method should meet

- precision, which means that the sum of allocated losses should be as close as possible (in the ideal case equal) to the total losses,
- respecting the value of bus power,
- balanced allocation of losses to generators and loads
- taking into account the relative position of buses in the system (the fact that loads close to generators contribute less to losses and vice versa should be reflected),
- minimizing arbitrariness,
- minimizing volatility,
- using a simple algorithm easy to understand and implement.

Let us emphasize here that in the following text a single-phase representation is used, i.e. the symbols of power in formulas represent single-phase power and power losses apply to only one phase.

### 3.1 PRO RATA ALLOCATION

This allocation method uses the following formulas

$$\begin{aligned}\Delta P_{Gi} &= \frac{1}{2} \Delta P \frac{P_{Gi}}{\sum_{i=1}^p P_{Gi}} \\ \Delta P_{Lj} &= \frac{1}{2} \Delta P \frac{P_{Lj}}{\sum_{j=1}^q P_{Lj}}\end{aligned}\tag{3.1}$$

where  $\Delta P_{Gi}$  real power losses allocated to generation at bus  $i$ ,

$\Delta P_{Lj}$  real power losses allocated to load at bus  $j$ ,

$\Delta P$  total real power losses of the system,

$p$  number of generators,

$q$  number of loads,

$n$  number of buses of the system ( $n = p + q$ ).

It is obvious from Equations (3.1) that losses allocated both to generators and loads are always positive. Also, it should be noted that losses are allocated to all system buses, including the slack bus.

### 3.2 PROPORTIONAL SHARING ALLOCATION

This method is actually an application of Kirchhoff's laws. It is based on an assumption that power flow entering a given bus from a certain branch is distributed among the branches drawing power from the bus in proportion to power flows of these branches, which is a hypothesis that can be neither proved nor disproved.

Let us summarize an approach published in [4]. Losses are first allocated to loads. A new quantity, called gross load (i.e. load in an ideal network where there are no losses), is defined

$$P'_L = P_L + \Delta P \quad (3.2)$$

$$P'_L = \sum_{j=1}^q P'_{Lj} \quad (3.3)$$

where  $P'_{Lj}$  gross load at bus  $j$ ,

$P'_L$  total gross load.

According to the proportional sharing principle, the power balance at every bus of an ideal network can be expressed as

$$P'_i = P_{Gi} + \sum_{j=1}^s c_{ji} P'_j \quad \text{for } i = 1, 2, \dots, n \quad (3.4)$$

$$c_{ji} = \frac{P'_{ji}}{P'_j} \approx \frac{P_{ji}}{P_j} \quad (3.5)$$

$$P'_i = P_{Gi} + \sum_{j=1}^s \frac{P_{ji}}{P_j} P'_j \quad (3.6)$$

where  $P'_i$  gross power injected at bus  $i$ ,

$P_{Gi}$  generation at bus  $i$ ,

$\sum c_{ji} P'_j$  power flow entering bus  $i$  from branches connected to it,

$s$  set of buses which power flows enter bus  $i$ ,

$P'_{ji}$  gross power flow from bus  $j$  to bus  $i$ ,

$P_{ji}$  actual power flow from bus  $j$  to bus  $i$  (measured at bus  $j$ ),

$P'_j$  gross power injected at bus  $j$ ,

$P_j$  actual power injected at bus  $j$ .

Equations (3.6) represent a set of nonlinear equations. Its solution yields the gross power injections. Once gross power injections are known, gross loads and losses allocated to loads can be calculated

$$P'_{L_j} = \frac{P'_j}{P_j} P_{L_j} \quad (3.7)$$

$$\Delta P_{L_j} = P'_{L_j} - P_{L_j} \quad (3.8)$$

The allocation of losses to generators is calculated in a similar way.

Also in this case, losses allocated both to generators and loads are always positive and they are allocated to all system buses, including the slack bus.

### 3.3 INCREMENTAL TRANSMISSION LOSS COEFFICIENT ALLOCATION

With this approach, losses are allocated to generators and loads by means of incremental transmission loss coefficients (the allocation of reactive power losses is not treated in this work, therefore only incremental real power loss coefficients are mentioned)

$$\begin{aligned} \frac{\partial \Delta P}{\partial P_i} &= \frac{\partial \Delta P}{\partial (P_{Gi} - P_{Li})} \\ \frac{\partial \Delta P}{\partial Q_i} &= \frac{\partial \Delta P}{\partial (Q_{Gi} - Q_{Li})} \end{aligned} \quad (3.9)$$

It should be noted that the existing methods of incremental transmission loss coefficient allocation neglect the influence of bus reactive powers on real power losses (see [11], [13], [14]). In author's opinion real as well as reactive power should be taken into account and he incorporated both incremental transmission loss coefficients in Equation (3.9) into the methods proposed in this chapter.

Incremental transmission loss coefficients express the dependence of a total power loss change on a bus real or reactive power change (note that the incremental transmission loss coefficients of the slack bus are equal to zero). Incremental transmission loss coefficients thus represent an indicator of sensitivity of the system power losses to power changes at a given bus, but they do not have any quantitative meaning. Loss allocation is therefore calculated by integrating incremental transmission loss coefficients over the corresponding real and reactive power

$$\begin{aligned}\Delta P_{P_i} &= \int_0^{P_i} \frac{\partial \Delta P}{\partial P_i} dP_i \\ \Delta P_{Q_i} &= \int_0^{Q_i} \frac{\partial \Delta P}{\partial Q_i} dQ_i\end{aligned}\tag{3.10}$$

The total losses allocated to bus  $i$  are given by the sum of losses allocated to real and reactive power

$$\Delta P_i = \Delta P_{P_i} + \Delta P_{Q_i}\tag{3.11}$$

However, the practical solution of incremental transmission loss coefficients is very complicated and thus simplifications are used. The problem is usually linearized. Theoretically, the sum of allocated losses should match the total losses. In reality – due to simplifications – they are not equal

$$\Delta P \neq \sum_{i=2}^n \Delta P_i\tag{3.12}$$

Therefore a *normalization* of allocated losses is performed. The normalization is a procedure that recalculates the allocated losses so that their sum matches the total losses

$$\Delta P_i^N = \Delta P_i \frac{\Delta P}{\sum_{i=2}^n \Delta P_i}\tag{3.13}$$

where  $\Delta P_i^N$  normalized real power losses allocated to bus  $i$ .

The incremental transmission loss coefficient methods may allocate negative losses to a certain bus, which means that thanks to the position of the bus in the system the generation or load at this bus contributes to decreasing transmission losses. These negative allocated losses can be interpreted as cross subsidies. In such a case the author proposes a procedure that he has called *positive loss allocation*. It assigns zero losses to all buses with negative allocated losses; the losses allocated to the other buses are recalculated proportionally to the ratio of the total losses and the sum of positive allocated losses

$$\Delta P_i^{NP} = \Delta P_i \frac{\Delta P}{\sum_{i=2}^{n_p} \Delta P_i} \quad \text{for } i = 2, 3, \dots, n_p \quad (3.14)$$

where  $\Delta P_i^{NP}$  normalized positive real power losses allocated to bus  $i$ ,  
 $n_p$  set of buses with positive allocated real power losses.

### 3.3.1 Incremental Transmission Loss Coefficient Allocation Based on Incremental Slack Bus Power Coefficient Solution

For a given system, the following conditions of power balance must be fulfilled

$$\sum_{i=1}^n P_i - \Delta P = 0 \quad \sum_{i=1}^n Q_i - \Delta Q = 0 \quad (3.15)$$

Equations (3.15) are differentiated with respect to  $P_i$  and  $Q_i$  for  $i = 2, 3, \dots, n$ . The results in a modified form are

$$\begin{aligned} \frac{\partial \Delta P}{\partial P_i} &= 1 + \frac{\partial P_1}{\partial P_i} & \frac{\partial \Delta P}{\partial Q_i} &= \frac{\partial P_1}{\partial Q_i} \\ \frac{\partial \Delta Q}{\partial P_i} &= \frac{\partial Q_1}{\partial P_i} & \frac{\partial \Delta Q}{\partial Q_i} &= 1 + \frac{\partial Q_1}{\partial Q_i} \end{aligned} \quad (3.16)$$

The solution of the problem consists in calculating the values of incremental slack bus power coefficients and consequently incremental transmission loss coefficients. It is described by the following matrix equation

$$\begin{bmatrix} \frac{\partial P_1}{\partial P_2} \\ \vdots \\ \frac{\partial P_1}{\partial P_i} \\ \vdots \\ \frac{\partial P_1}{\partial P_n} \\ \frac{\partial P_1}{\partial P_1} \\ \frac{\partial Q_2}{\partial P_1} \\ \vdots \\ \frac{\partial P_1}{\partial Q_i} \\ \vdots \\ \frac{\partial P_1}{\partial P_1} \\ \frac{\partial Q_n}{\partial P_1} \end{bmatrix} = \begin{bmatrix} \frac{\partial \delta_2}{\partial P_2} & \dots & \frac{\partial \delta_i}{\partial P_2} & \dots & \frac{\partial \delta_n}{\partial P_2} & \frac{\partial V_2}{\partial P_2} & \dots & \frac{\partial V_i}{\partial P_2} & \dots & \frac{\partial V_n}{\partial P_2} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{\partial \delta_2}{\partial P_i} & \dots & \frac{\partial \delta_i}{\partial P_i} & \dots & \frac{\partial \delta_n}{\partial P_i} & \frac{\partial V_2}{\partial P_i} & \dots & \frac{\partial V_i}{\partial P_i} & \dots & \frac{\partial V_n}{\partial P_i} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \delta_2}{\partial P_n} & \dots & \frac{\partial \delta_i}{\partial P_n} & \dots & \frac{\partial \delta_n}{\partial P_n} & \frac{\partial V_2}{\partial P_n} & \dots & \frac{\partial V_i}{\partial P_n} & \dots & \frac{\partial V_n}{\partial P_n} \\ \frac{\partial P_n}{\partial \delta_2} & \dots & \frac{\partial P_n}{\partial \delta_i} & \dots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_2} & \dots & \frac{\partial P_n}{\partial V_i} & \dots & \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_2}{\partial \delta_2} & \dots & \frac{\partial Q_2}{\partial \delta_i} & \dots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial V_2} & \dots & \frac{\partial Q_2}{\partial V_i} & \dots & \frac{\partial Q_2}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{\partial \delta_2}{\partial Q_i} & \dots & \frac{\partial \delta_i}{\partial Q_i} & \dots & \frac{\partial \delta_n}{\partial Q_i} & \frac{\partial V_2}{\partial Q_i} & \dots & \frac{\partial V_i}{\partial Q_i} & \dots & \frac{\partial V_n}{\partial Q_i} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \delta_2}{\partial Q_n} & \dots & \frac{\partial \delta_i}{\partial Q_n} & \dots & \frac{\partial \delta_n}{\partial Q_n} & \frac{\partial V_2}{\partial Q_n} & \dots & \frac{\partial V_i}{\partial Q_n} & \dots & \frac{\partial V_n}{\partial Q_n} \end{bmatrix} \begin{bmatrix} \frac{\partial P_1}{\partial \delta_2} \\ \vdots \\ \frac{\partial P_1}{\partial \delta_i} \\ \vdots \\ \frac{\partial P_1}{\partial \delta_n} \\ \frac{\partial P_1}{\partial P_1} \\ \frac{\partial \delta_n}{\partial P_1} \\ \frac{\partial V_2}{\partial P_1} \\ \vdots \\ \frac{\partial P_1}{\partial V_i} \\ \vdots \\ \frac{\partial P_1}{\partial P_1} \\ \frac{\partial V_n}{\partial P_1} \end{bmatrix} \quad (3.17)$$

The partial derivatives of slack bus power with respect to bus voltage magnitudes and phase angles are calculated as

$$\frac{\partial P_1}{\partial \delta_i} = V_1 V_i (G_{i1} \sin \delta_{li} - B_{i1} \cos \delta_{li}) \quad (3.18)$$

$$\frac{\partial P_1}{\partial V_i} = V_1 (G_{i1} \cos \delta_{li} + B_{i1} \sin \delta_{li}) \quad (3.19)$$

The solution of the partial derivatives of bus voltage phase angles and magnitudes with respect to bus real and reactive power consists in a complicated solution of a set of nonlinear equations. Still, we can simplify it by the following assumption. First, let us assume that the real power at bus 2 changes by  $dP_2$  while reactive power  $Q_2$  and the real and reactive power at the other buses remain constant (except the slack bus), which leads to the solution of the partial derivatives of bus voltage phase angles and magnitudes with respect to variable  $P_2$ . Similarly the partial derivatives with respect to variable  $P_3$  can be calculated etc. Then we can proceed to the calculation of incremental slack bus power coefficients

The allocation of losses according to Equation (3.10) cannot be done since it is impossible to find an analytical formula for incremental transmission loss coefficients that can be integrated. Thus a linear approach is applied.



### 3.3.2 Incremental Transmission Loss Coefficient Allocation Based on the Bus Impedance Matrix

The second proposed approach is based on incremental transmission loss coefficient solution using the bus impedance matrix. To simplify the solution, let the shunt admittances be neglected.

If the following coefficients are used

$$A_{ij} = \frac{R_{ij}}{V_i V_j} \cos \delta_{ij} \quad B_{ij} = \frac{R_{ij}}{V_i V_j} \sin \delta_{ij} \quad (3.20)$$

$$G_{ij} = \frac{X_{ij}}{V_i V_j} \cos \delta_{ij} \quad H_{ij} = \frac{X_{ij}}{V_i V_j} \sin \delta_{ij}$$

incremental loss coefficients can be expressed as

$$\begin{aligned} \frac{\partial \Delta P}{\partial P_i} &= 2 \sum_{j=2}^n A_{ij} P_j - 2 \sum_{\substack{j=2 \\ j \neq i}}^n B_{ij} Q_j \\ \frac{\partial \Delta P}{\partial Q_i} &= 2 \sum_{j=2}^n A_{ij} Q_j + 2 \sum_{\substack{j=2 \\ j \neq i}}^n B_{ij} P_j \end{aligned} \quad (3.21)$$

and, consequently, allocated losses as

$$\begin{aligned} \Delta P_{P_i} &= A_{ii} P_i^2 + 2 \sum_{\substack{j=2 \\ j \neq i}}^n (A_{ij} P_j - B_{ij} Q_j) P_i \\ \Delta P_{Q_i} &= A_{ii} Q_i^2 + 2 \sum_{\substack{j=2 \\ j \neq i}}^n (A_{ij} Q_j + B_{ij} P_j) Q_i \end{aligned} \quad (3.22)$$

If a simplified – linear – approach is applied, losses are allocated the following way

$$\Delta P_{P_i} = \frac{\partial \Delta P}{\partial P_i} P_i$$

$$\Delta P_{Q_i} = \frac{\partial \Delta P}{\partial Q_i} Q_i$$

### 3.4 SUPERPOSITION ALLOCATION

It is based on a simple hypothesis that the total influence of all buses of the system on the total losses is the superposition of the individual influence of buses. Similarly to the proportional sharing principle, it is an idea that is neither provable nor disprovable.

The superposition method compares the total losses of the system with the original load distribution and the total losses of the system where the influence of generation or load at bus  $i$  has been eliminated (i.e. the real and reactive power at this bus has been set to zero). According to the hypothesis, losses allocated to bus  $i$  are given by the difference of these two values

$$\Delta P_i = \Delta P - \Delta P_{i0} \quad (3.23)$$

where  $\Delta P_i$  losses allocated to bus  $i$ ,

$\Delta P$  total losses of the system with the original load distribution,

$\Delta P_{i0}$  total losses of the system where the power at bus  $i$  has been eliminated.

Also in this case, it can be expected that the sum of allocated losses will not match the total losses and therefore the normalization procedure should be applied. It can be also expected that the procedure may allocate negative losses and then positive allocation – if required – can be used.

## 4 PRACTICAL APPLICATION

The allocation methods treated in Chapter 3 have been tested on three sample systems

- five-bus bus sample system [47],
- IEEE fourteen-bus sample system [40],
- IEEE twenty-three-bus sample system [40].

First, load flow solution is performed. The modified Newton-Raphson method using voltage-dependent load characteristics has been applied. The results are then used as the input data for loss allocation. Five allocation methods have been tested. Two of them are already existing methods: pro rata allocation and proportional sharing allocation. The other three are the methods defined by the author. In case of the incremental method based on the bus impedance matrix both the integration and linear approach have been applied. For each sample system, calculations have been done for a case where the dependence of load on voltage is taken into account as well as for a system with constant loads. Note that if voltage-dependent loads are taken into account, the given load real and reactive power represent the values at nominal voltage.

All simulations have been done by using Matlab programming language except for forming the bus admittance matrices done in the Excel software.

The following abbreviations and symbols are used in the text

- *PR* – pro rata allocation,
- *PS* – proportional sharing allocation,
- *INC I* – incremental allocation (slack bus),
- *INC II* – incremental allocation (impedance matrix),
- *SUP* – superposition allocation,
- $\Delta P$  – total losses,
- $\Delta P_i$  – losses allocated to bus  $i$ ,
- $\Delta P_i^N$  – normalized losses allocated to bus  $i$ ,
- $\Delta P_i^{NP}$  – normalized positive losses allocated to bus  $i$ .

#### **4.1 FIVE-BUS SAMPLE SYSTEM**

There is only a small difference between the results of load flow solution and the allocation of losses for voltage-dependent and constant loads, which is in accordance with the declared effect of the modified Newton-Raphson algorithm.

The first of the incremental methods shows poor precision, relative error (defined as the difference between the sum of allocated losses and the total losses in percent of the total losses) is 645.3 % for voltage-dependent loads and 638.4 % for constant loads. Also, the distribution of allocated losses is significantly different from the one for the other incremental approach. The incremental method based on the impedance matrix provides much more precise results, surprisingly better in case of the simplified linear approach (relative error, as defined above, is –53.7 % both for voltage-dependent and constant loads, while in case of the integration approach it represents –78.2 % and –78.1 % for voltage-dependent and constant loads respectively). This phenomenon could be attributed to the fact that the incremental method using the bus impedance matrix involves a simplification that consists in neglecting the shunt admittances.

Superposition allocation provides apparently quite precise results (i.e. the sum of allocated losses is relatively close to the total losses), but the distribution of allocated losses is very different from the one of the other methods.

#### **4.2 FOURTEEN-BUS SAMPLE SYSTEM**

Also in case of the fourteen-bus sample system only a small difference can be found between the results of load flow solution and the allocation of losses for voltage-dependent and constant loads.

The first of the incremental methods shows again poor precision, relative error is 584.8 % for voltage-dependent loads and 561.0 % for constant loads. The results of the incremental method based on the impedance matrix are much more precise, with almost same precision for the integration and linear approach. We should point out, however, that while the linear approach results in the over-recovery of losses, the

sum of allocated losses for the integration approach is less than the total losses. Also, it should be noted that this incremental method allocates more losses to generators than to loads. In case of the linear approach 81.8 % and 82.4 % of the total losses is allocated to generators for voltage-dependent and constant loads respectively. The values for the integration approach are even higher: 98.8 % (voltage-dependent loads) and 99.0 (constant loads). This phenomenon could be partially limited by the application of positive allocation where the losses allocated to generators decrease to about 65 % of the total losses.

As superposition allocation is concerned, it provides extremely volatile results, with a relative error of  $-7669.0$  % (!) for voltage-dependent loads, while in case of constant load the relative error is equal only to  $-0.6$  %.

### **4.3 TWENTY-THREE-BUS SAMPLE SYSTEM**

The difference between the results of load flow solution and the allocation of losses for voltage-dependent and constant loads is – analogically to the two previous sample systems – very small.

The first of the incremental methods shows now even poorer results, relative error is  $-1514.2$  % for voltage-dependent loads and  $1538.4$  % for constant loads. The results of the incremental method based on the impedance matrix are again much more precise, with excellent results for the integration approach (relative error of  $8.5$  % for voltage-dependent loads and  $11.6$  % for constant loads). Also in case of the twenty-three-bus sample system, the incremental method based on the bus impedance matrix allocates more losses to generators than to loads. In case of the linear approach  $90.1$  % and  $89.2$  % of the total losses is allocated to generators for voltage-dependent and constant loads respectively. In the integration approach, the losses allocated to generator are even higher than the total losses:  $103.9$  % (voltage-dependent loads) and  $101.3$  (constant loads) of the total losses. The application of positive allocation limits these numbers to about  $70$  % for the linear approach and  $60$  % for the integration approach.

Superposition allocation results in completely inconsistent values with the negative sum of allocated losses.

## 5 CONCLUSIONS

The primary goal of the dissertation was to find a new allocation method using incremental transmission loss coefficients. In fact, two allocating methods of this type have been proposed. In both cases the influence of the bus reactive power on the system losses has been incorporated to allocation algorithms, which is a novel approach because the role of the bus reactive power is neglected in the existing incremental methods.

The first of the incremental methods defined by the author is based on incremental slack bus power coefficient solution. The allocation of losses in this case can be done only for a simplified linear approach because it is impossible to find an analytical formula for incremental transmission loss coefficients that could be integrated. The second method uses incremental transmission loss coefficient allocation based on the bus impedance matrix. The allocation of losses has been calculated by integrating incremental transmission loss coefficients over the corresponding real and reactive power and, for comparison, also a linear approach has been used. In addition to the incremental methods, another method called superposition allocation has been proposed. It is based on the hypothesis of the linear superposition of the individual influence of buses on the losses.

A secondary goal of the dissertation was to incorporate voltage-dependent load characteristics in load flow solution. A modification of the Newton-Raphson algorithm using the exponential load model has been defined by the author. This modified method has been then applied to load flow solution. The aim of the modification was to make load flow solution more precise. Nevertheless, the author would like to emphasize that in case of a real system it would be necessary to analyze the availability and precision (or fidelity) of the input data, i.e. parameters  $x_i, y_i$ .

The three proposed methods together with two other known approaches (pro rata allocation and proportional sharing allocation) have been tested on three sample systems (five, fourteen a twenty-three buses), in each case both for voltage-dependent load characteristics and constants loads.

The primary criterion for assessing an allocation method is its precision, which means that the sum of allocated losses should be as close as possible (in the ideal case equal) to the total losses. From the definition of pro rata and proportional sharing algorithms in Chapter 3.1. and 3.2. respectively, it is clear that in these methods the sum of allocated losses matches the total losses. An overall comparison of relative error (difference between the sum of allocated losses and the total losses in percent of the total losses) for the other methods can be found in Table 5.1.

**Table 5.1. Relative Error of Allocation Methods**

Method	5-Bus System		14-Bus System		23-Bus System	
	Volt. Dep.	Const.	Volt. Dep.	Const.	Volt. Dep.	Const.
	Loads [%]	Loads [%]	Loads [%]	Loads [%]	Loads [%]	Loads [%]
INC I	645.3	638.4	584.8	561.0	-1514.2	-1538.4
INC II (Lin.)	-53.7	-53.7	22.6	23.9	146.5	147.3
INC II (Int.)	-78.2	-78.1	-23.0	-21.9	8.5	11.6
SUP	-58.0	39.7	-7669.0	-0.6	-194.0	-170.0

From the results of simulations, the following conclusions can be drawn.

- It is obvious from extreme volatility of superposition allocation that the hypothesis of superposition in case of losses cannot be confirmed.
- Incremental transmission loss coefficient allocation based on incremental slack bus power coefficient solution shows worse results. High volatility of this allocation method can be probably attributed to the used simplification consisting in the application of linear approach.
- Out of the two incremental methods, the one using the bus impedance matrix should be preferred. Two sets of results for the integration and linear approach are available. It can be concluded from the results that the precision of allocation (as defined above) for the integration allocation increases with the increasing number of buses in the system. This phenomenon should be, however, verified by testing in large systems. Please note that also for incremental method using the bus impedance matrix a simplification has been assumed (neglecting the shunt admittances), which could explain the errors of this method as well as the fact that in case of the smallest system the simplified linear approach provides better results (from the point of view of the precision of allocation).

It is thus obvious that out of the three allocation methods defined by the author, only one can be really taken into account for practical application.

It should be reiterated here that it is not the aim of this dissertation to take a final decision on which of the methods should be chosen. The task was to find a utilizable allocation method using incremental transmission loss coefficients, which has been fulfilled. Still, the results of simulations should serve as important data for selecting an appropriate allocation method for practical application.

Let us now discuss the advantages and disadvantages of incremental transmission loss coefficient allocation based on the bus impedance matrix from the point of view of the experimental results and compare it with pro rata and proportional sharing allocation. The following can be stated

- Pro rata allocation provides a very simple algorithm, but it does not take into account the relative position of buses in the network. It should be, however, considered if it is really fair that the allocation of losses takes into account the relative position of buses in the system. In the opinion of the author it is questionable because the user at a given bus cannot affect its situation (position). Also, the pro rata method allocates losses to all buses, including the slack bus, which can be considered as a problematic issue.
- Proportional sharing allocation provides a loss distribution that is supposed to be a compromise between the pro rata and incremental approach. Since the algorithm of this method is based on an assumption that cannot be proved, its fairness can be questioned and in case of a practical application, it would probably be hardly acceptable for the participants in the electricity market.
- Incremental transmission allocation is strongly dependent on the selection of the slack bus. Finally, we have seen that the incremental method tends to allocate more than 50 % of the losses to generators. This could be limited by the application of positive allocation, but it would mean a significant arbitrary intervention into the method.

To sum up, it is obvious that loss allocation represents an important technical problem and its solution remains open. We have seen that all methods contain arbitrary elements and that the results of allocation depend significantly on used method. The final acceptance or refusal depends, however, on the participants in the electricity market – users of the transmission system.

Nevertheless, the author would like to emphasize that the theoretical and experimental results obtained in this dissertation could be used as a basis for deciding on an appropriate allocation method for practical application. It is clear, however, that conclusions apply to relatively small systems and therefore further research should follow. The future research activities should concentrate on the following areas

- testing in larger systems,
- incorporating voltage-controlled buses in load flow solution and loss allocation,
- analysis of the dependence of results on the selection of the slack bus,
- analysis of the role of reactive power in the transmission system,
- analysis of economic consequences of loss allocation,
- analysis of other aspects of transmission system operation (congestion management, system services etc.).

## 6 REFERENCES

- [1] Andersson G.: *Modelling and Analysis of Electric Power Systems*. Lecture notes. Swiss Federal Institute of Technology, Zurich 2003. Available: <http://www.eeh.ee.ethz.ch/downloads/academics/courses/35-526.pdf>.
- [2] Arrillaga J., Arnold C. P., Harker B. J.: *Computer Modelling of Electrical Power Systems*. John Wiley & Sons, London 1983.
- [3] Arrillaga J., Arnold C. P.: *Computer Analysis of Power Systems*. John Wiley & Sons, London 1990.
- [4] Bialek J.: Tracing the Flow of Electricity. *IEE Proceedings – Generation, Transmission and Distribution*, vol. 143, July 1996, pp. 313-320.
- [5] Bialek J.: Allocation of Transmission Supplementary Charge to Real and Reactive Loads. *IEEE Transactions on Power Systems*, vol. 13, no. 3, August 1998, pp. 749-754.
- [6] Bjorgan R., Song H., Liu C.-C., Dahlgren R.: Pricing Flexible Electricity Contracts. *IEEE Transactions on Power Systems*, vol. 15, no. 2, May 2000, pp. 477-482.
- [7] Chmela M., Haluzík E.: Applying Voltage Dependent Load Characteristics in Incremental Transmission Loss Solution. In *Control of Power System 2000*. Proceedings of the international conference held on June 15 and 16, 2000 in Bratislava, pp. 110-115.
- [8] Chmela M., Haluzík E.: Load Flow Solution Using Voltage Dependent Load Characteristics. In *Elektroenergetika 2000*. Proceedings of the international conference held in 2000 in Ostrava, pp. 125-128.
- [9] Chmela M., Haluzík E.: Alokace ztrát v přenosové soustavě. In *Současnost elektroenergetiky 2003*. Proceedings of the conference held by the Brno University of Technology on May 19 and 20, 2003 in Brno, pp. 72-77.
- [10] Conejo A. J., Galiana F. D., Kockar I.: Z-Bus Loss Allocation. *IEEE Transactions on Power Systems*, vol. 16, no. 1, February 2001, pp. 105-109.
- [11] Conejo A. J., Arroyo J. M., Alguacil N., Guijarro A. L.: Transmission Loss Allocation: A Comparison of Different Practical Algorithms. *IEEE Transactions on Power Systems*, vol. 17, no. 3, August 2002, pp. 571-576.
- [12] Costa V. M., Martins N., Pereira J. L. R.: Developments in the Newton Raphson Power Flow Formulation Based on Current Injections. *IEEE Transactions on Power Systems*, vol. 14, no. 4, November 1999, pp. 1320-1326.
- [13] Galiana F. D., Phelan M.: Allocation of Transmission Losses to Bilateral Contracts in a Competitive Environment. *IEEE Transactions on Power Systems*, vol. 15, no. 1, February 2000, pp. 143-150.
- [14] Galiana F. D., Conejo A. J., Kockar I.: Incremental Transmission Loss Allocation Under Pool Dispatch. *IEEE Transactions on Power Systems*, vol. 17, no. 1, February 2002, pp. 26-33.



- [15] Gellings C. W.: Power Delivery System of the Future. *IEEE Power Engineering Review*, December 2002, pp. 7-12.
- [16] Gómez Expósito A., Riquelme Santos J. M., González García T., Ruiz Velasco E. A.: Fair Allocation Of Transmission Losses. *IEEE Transactions on Power Systems*, vol. 15, no. 1, February 2000, pp. 184-188.
- [17] Grieger V., Gramblička M., Novák M., Pokorný M.: *Prevádzka, riadenie a kontrola prepojenej elektrizačnej sústavy*. Žilinská univerzita v Žiline, Žilina 2001.
- [18] Gross G., Tao S.: A Physical-Flow-Based Approach to Allocating Transmission Losses in a Transaction Framework. *IEEE Transactions on Power Systems*, vol. 15, no. 2, May 2000, pp. 631-637.
- [19] Gubina F., Grgič D., Banič I.: A Method for Determining the Generator's Share in a Consumer Load. *IEEE Transactions on Power Systems*, vol. 15, no. 4, November 2000, pp. 1376-1381.
- [20] Guile A. E., Paterson W.: *Electrical Power Systems Volume One*. Oliver & Boyd, Edinburgh 1969.
- [21] Haluzík E., Pavlinec P.: Optimalizace výroby jalové energie z existujících zdrojů v ustáleném stavu elektrizační soustavy – I. In *Nové přístupy k navrhování elektrických sítí*. Proceedings of the workshop held by the EGÚ Brno Research Institute in 1984 in Brno, pp. 179-193.
- [22] Haluzík E., Pavlinec P.: Optimalizace výroby jalové energie z existujících zdrojů v ustáleném stavu elektrizační soustavy – II. In *Nové přístupy k navrhování elektrických sítí*. Proceedings of the workshop held by the EGÚ Brno Research Institute in 1984 in Brno, pp. 194-205.
- [23] Haluzík E.: *Řízení provozu elektrizačních soustav*. 2<sup>nd</sup> ed., Vysoké učení technické v Brně, Brno 1987.
- [24] Haluzík E., Pavlinec P., Haluzíková A.: Výpočet poměrného přírůstku výkonu bilančního uzlu a jeho použití při řešení úlohy hospodárného rozdělování výroby výkonů. In *Zborník prác z 2. Vedeckej konferencie Elektrotechnickej fakulty SVŠT*, Bratislava, 1989, pp. 157-161.
- [25] Haluzík E., Chmela M.: Výpočet poměrných přírůstků ztrát. *Časopis EE*, no. 4, 1999, pp. 9-10.
- [26] Heřman J. et al.: *Příručka silnoproudé elektrotechniky*. SNTL, Praha 1984.
- [27] Horák K.: *Výpočet elektrických sítí*. SNTL, Praha 1980.
- [28] Jež et al.: Očekávaný stav a provoz ES ČR v roce 2002 a navazující perspektivě. In *Očekávaný stav a provoz ES ČR a způsob obchodování elektrinou na úrovni roku 2002*. Proceedings of the workshop held by the EGÚ Brno Research Institute on September 18 and 19, 2001 in Brno.

- [29] Kočenda E., Čábelka Š.: Liberalization in the Energy Sector: Transition and Growth. *Osteuropa Wirtschaft*, 44, 1999, 1, pp. 104-116.
- [30] Kolcun M. et al.: *Riadenie prevádzky elektrizačnej sústavy*. Mercury-Smékal, 2002 Košice.
- [31] Lamoureux M. A.: Evolution of Electric Utility Restructuring in the UK. *IEEE Power Engineering Review*, June 2001, pp. 3-9, 35.
- [32] Lamoureux M. A.: U.S. Electric Energy Policy. *IEEE Power Engineering Review*, July 2002, pp. 12-17.
- [33] Lindén K., Segerqvist I.: *Modelling of Load Devices and Studying Load/System Characteristics*. Technical report. Chalmers University of Technology, Göteborg 1992. Available: <http://www.elteknik.chalmers.se/Publikationer/EKS.publ/Abstract/131L.pdf>.
- [34] Lyons P. K.: *EU Energy Policies towards the 21<sup>st</sup> Century*. EC Inform, 1998 Elstead.
- [35] Pan J., Teklu Y., Rahman S., Jun K.: Review of Usage-Based Transmission Cost Allocation Methods under Open Access. *IEEE Transactions on Power Systems*, vol. 15, no. 4, November 2000, pp. 1218-1224.
- [36] Park Y., Park J., Lim J., Won J.: An Analytical Approach for Transaction Costs Allocation in Transmission Systems. *IEEE Transactions on Power Systems*, vol. 13, no. 4, November 1998, pp. 1407-1412.
- [37] Pavlinec P.: Poplatky za přenosové a distribuční služby v tržním modelu ES ČR, metody a očekávaný reálný stav. In *Očekávaný stav a provoz ES ČR a způsob obchodování elektrinou na úrovni roku 2002*. Proceedings of the workshop held by the EGÚ Brno Research Institute on September 18 and 19, 2001 in Brno.
- [38] Pavlíček Z.: *Přenos a rozvod elektrické energie IIb*. Vysoké učení technické v Brně, Brno 1975.
- [39] Pérez-Arriaga I. J., Perán Montero F., Rubio Odériz F. J.: *Benchmark of Electricity Transmission Tariffs*. Report prepared for the Directorate-General for Energy and Transport / European Commission. Universidad Pontificia Comillas, Madrid 2002. Available: [http://europa.eu.int/comm/energy/electricity/publications/doc/bench\\_trans\\_tarif\\_en.pdf](http://europa.eu.int/comm/energy/electricity/publications/doc/bench_trans_tarif_en.pdf).
- [40] Ptáček J.: *Graphic Load Flow (GLF), Analýzy elektrických sítí (AES)*. Software manual. EGÚ Brno, 1997.
- [41] Rau N. S.: Transmission Loss and Congestion Cost Allocation – An Approach Based on Responsibility. *IEEE Transactions on Power Systems*, vol. 15, no. 4, November 2000, pp. 1401-1409.
- [42] Reiss L., Malý K., Pavlíček Z., Bizík J.: *Teoretická elektroenergetika II*. ALFA, 1979 Bratislava.
- [43] Rektorys K. et al.: *Přehled užití matematiky I*. 6<sup>th</sup> ed., Prometheus, Praha 1995.
- [44] Rektorys K. et al.: *Přehled užití matematiky II*. 6<sup>th</sup> ed., Prometheus, Praha 1995.

- [45] Rubio-Odériz F. J., Pérez-Arriaga I. J.: Marginal Pricing of Transmission Services: A Comparative Analysis of Network Cost Allocation Methods. *IEEE Transactions on Power Systems*, vol. 15, no. 1, February 2000, pp. 448-454.
- [46] Schwarz J., Staschus K., Knop T., Zettler K.-R.: Overview of the EU Electricity Directive. *IEEE Power Engineering Review*, April 2000, pp. 4-7.
- [47] Stagg W. G., El-Abiad A. H.: *Computer Methods in Power System Analysis*. McGraw-Hill, New York 1968.
- [48] Staschus K.: German Open Access and Transmission Pricing within the European Framework. *IEEE Power Engineering Review*, April 2000, pp. 17-18.
- [49] Šolc P., Toufar J.: Podpůrné a systémové služby v ES ČR – potřeby a úhrady. In *Očekávaný stav a provoz ES ČR a způsob obchodování elektřinou na úrovni roku 2002*. Proceedings of the workshop held by the EGÚ Brno Research Institute on September 18 and 19, 2001 in Brno.
- [50] Tao S., Gross G.: Transmission Loss Compensation in Multiple Transaction Networks. *IEEE Transactions on Power Systems*, vol. 15, no. 3, August 2000, pp. 909-915.
- [51] Tomec A.: Otevření trhu a vazba mezi provozovatelem přenosové soustavy a operátorem trhu. In *Očekávaný stav a provoz ES ČR a způsob obchodování elektřinou na úrovni roku 2002*. Proceedings of the workshop held by the EGÚ Brno Research Institute on September 18 and 19, 2001 in Brno.
- [52] Tradacete Cocera A.: The Role of EC Competition Policy in the Liberalisation of EU Energy Markets. Proceedings of the *European Energy Millenium Forum* held on April 27, 2000 in Brussels.  
Available: [http://europa.eu.int/comm/competition/speeches/text/sp2000\\_003\\_en.pdf](http://europa.eu.int/comm/competition/speeches/text/sp2000_003_en.pdf).
- [53] Unsihuy C., Saavedra O. R.: Métodos para la asignación de pérdidas de transmisión en mercados eléctricos competitivos. In *XIV – Congresso Brasileiro de Automática*. Proceedings of the congress held on September 2 to 5, 2002 in Natal, pp. 2598-2604.
- [54] Van Roy P., Belmans R., Pepermans G., Proost S., Willems B., Conings L.: *Opening of the European Market for Electricity*. University of Leuven Energy Institute, Leuven 2000.
- [55] Ward A. G., Watson N. R., Arnold C. P., Turner A. J., Ring B. J.: Inversion of Real Time Spot Prices in the Direction of Real Power Flow in a Transmission Line. *IEEE Transactions on Power Systems*, vol. 15, no. 4, November 2000, pp. 1197-1203.
- [56] Weedy B. M.: *Electrical Power Systems*. 3<sup>rd</sup> ed., John Wiley & Sons, London 1979.
- [57] Xie K., Song Y.-H., Stonham J., Yu E., Liu G.: Decomposition Model and Inferior Point Methods for Optimal Spot Pricing of Electricity in Deregulation Environments. *IEEE Transactions on Power Systems*, vol. 15, no. 1, February 2000, pp. 39-50.
- [58] Yu C. W., David A. K.: Pricing Transmission Services in the Context of Industry Deregulation. *IEEE Transactions on Power Systems*, vol. 12, 1997, no. 1, pp. 503-510.

- [59] *Annual Report 2002*. ČEPS, Praha 2003. Available: <http://www.ceps.cz/dokumenty/ceps02.pdf>.
- [60] *Directive 96/92/EC of the European Parliament and of the Council of 19 December 1996 concerning common rules for the internal market in electricity*. Official Journal of the European Communities L 027, January 30, 1997, pp. 0020-0029. Available: [http://europa.eu.int/smartapi/cgi/sga\\_doc?smartapi!celexapi!prod!CELEXnumdoc&lg=EN&numdoc=31996L0092&model=guichett](http://europa.eu.int/smartapi/cgi/sga_doc?smartapi!celexapi!prod!CELEXnumdoc&lg=EN&numdoc=31996L0092&model=guichett).
- [61] *Kodex přenosové soustavy*. ČEPS, Praha 2003. Available: <http://www.ceps.cz/detail.asp?cepsmenu=5&IDP=61&PDM2=0&PDM3=0&PDM4=0>.
- [62] *Review of Transmission Loss Adjustment Factors*. Report of the ESB National Grid. ESB, Dublin 2003. Available: <http://www.eirgrid.com/EirGridPortal/uploads/General%20Documents/160403%20Derivation%20of%20TLAFs%20ver2.pdf>.
- [63] *Using MATLAB (Version 6)*. Software manual. The MathWorks, Inc., Natick 2000.
- [64] *Zákon č. 458/2000 Sb., o podmínkách podnikání a o výkonu státní správy v energetických odvětvích a o změně některých zákonů (energetický zákon)*. Sbírka zákonů Česká republiky. Available: [http://www.eru.cz/zak\\_458\\_2000.doc](http://www.eru.cz/zak_458_2000.doc).

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English, French, Russian, Spanish

## **ABSTRACT**

The dissertation deals with the allocation of losses in the transmission system. The transmission losses are nonlinear functions of power flows in the lines of the system and thus it is not possible to associate directly a given user (generator or load) with a corresponding part of the transmission losses. However, a procedure that allows assigning the responsibility of users for a part of the losses must be applied. This process is called transmission loss allocation.

The dissertation focuses on allocation methods using incremental transmission loss coefficients. Two allocating methods of this type are proposed. In both cases the influence of the bus reactive power on the system losses has been incorporated to allocation algorithms, which is a novel approach because the role of the bus reactive power is neglected in the existing incremental methods.

The first of the incremental methods defined by the author is based on incremental slack bus power coefficient solution. The second method uses incremental transmission loss coefficient allocation based on the bus impedance matrix.

In addition to the incremental methods, another method called superposition allocation is proposed. It is based on the hypothesis of the linear superposition of the individual influence of buses on the losses.

The dissertation includes also a modification of the Newton-Raphson algorithm that incorporates voltage-dependent load characteristics in load flow solution.

The three proposed methods together with two other known approaches (pro rata allocation and proportional sharing allocation) are tested on three sample systems (five, fourteen a twenty-three buses), in each case both for voltage-dependent load characteristics and constants loads.

In the final part of the dissertation, the results of simulation are then discussed from the point of view of the possible practical application of the treated methods.

## ABSTRAKT

Disertační práce se zabývá problematikou alokace ztrát v přenosové soustavě. Tyto ztráty jsou nelineárními funkcemi toků výkonu ve vedeních soustavy a nelze tudíž přímo přiřadit odpovídající část ztrát danému uživateli přenosové soustavy. Přesto je však nutné ztráty v soustavě nějakým způsobem mezi její uživatele rozdělit. Tento proces se nazývá alokace ztrát.

Disertační práce se zaměřuje na alokační metody využívající poměrných přírůstků ztrát. V práci jsou navrženy dvě alokační metody tohoto typu. V obou případech byl vliv jalového výkonu v uzlech zapracován do alokačních algoritmů, což představuje vylepšení oproti stávajícím přírůstkovým metodám, kde se role jalového výkonu zanedbává.

První z přírůstkových metod navržených autorem práce je založena na výpočtu poměrného přírůstku výkonu bilančního uzlu. Druhá metoda pak využívá výpočet poměrných přírůstků ztrát pomocí impedanční uzlové matice.

K těmto dvěma přírůstkovým metodám byla v práci navržena ještě metoda třetí, nazvaná autorem superpoziční alokace. Je založena na předpokladu lineární superpozice vlivu výkonů v jednotlivých uzlech na ztráty.

Součástí práce je i návrh modifikovaného Newton-Raphsonova algoritmu, který respektuje napět'ovou závislost odběrů.

Tři navržené metody byly spolu s dvěma dalšími známými přístupy (poměrná alokace a alokace založená na poměrném rozdělení) testovány ve třech modelových sítích (o pěti, čtrnácti a dvaceti třech uzlech), vždy jak pro napět'ově závislé, tak pro konstantní odběry.

Závěrečnou část práce pak tvoří rozbor výsledků simulací z pohledu možného praktického využití jednotlivých alokačních metod.