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FULL-WAVE FINITE-ELEMENT ANALYSIS OF GENERAL MICROWAVE WAVEGUIDES

Teze Habilitační Práce

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Contents

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Since 1993, Zbyněk Raida is with the Institute of Radio Electronics of TU Brno, where he acts as a lecturer. He is engaged in teaching courses of electromagnetics, CAD for microwaves and antennas, and object oriented programming. Moreover, his research interests include numerical methods in electromagnetics, microwaves and antennas, application of artificial intelligence and optimization techniques.

Introduction

Growing significance of information in the development of human society is one of the main features of the 20th century. Therefore, more and more attention has to be paid to the transmission and to the processing of information. This fact causes that more and more powerful processors are produced, memories of higher and higher capacity are developed, wider and wider information channels are arranged.

Dealing with information channels, one of ways making them wider consists in shifting transmission of information to higher frequencies. Today’s commercial applications work on frequencies around 20 and 30 GHz, specific European systems are developed at 60 GHz. Therefore, the attention is turned to the transmission in the frequency range of micro- and millimeter-waves, to the analysis and design of circuits, antennas and transmission lines working at these frequencies, to the propagation of micro- and millimeter-waves in the atmosphere etc.

Let us concentrate on the analysis of the micro- and millimeter-wave transmission lines. In the lower part of the corresponding frequency range, the approximate quasi-static methods can be used whereas in its upper part, the full-wave dynamic methods have to be explored.

The quasi-static methods come from the assumption that the dominant mode of the wave propagating longitudinally along the transmission line is well approximated by a TEM wave. Then, the transversal fields are very close to the static ones and they can be derived from a static potential solution of Laplace's equation. The equation can be solved by the modified conformal mapping [1], by the finite-difference [1] or finite-element methods [13] [14], by the use of a variational expression or an integral equation.

Since electromagnetic fields supported by micro- and millimeter-wave transmission lines have longitudinal components which are no more negligible at higher frequencies, fields have to be represented by a combination of TE and TM waves and described by a vectorial wave equation which is the initial equation of the full-wave dynamic methods [1]. The vectorial wave equation can be solved by the finite-difference [1] or finite-element methods [14] or by the use of an analytical model of the transmission line [4].

If an analysis of a transmission line is going to be performed in the whole micro- and millimeter-wave frequency range, full-wave dynamic methods have to be used because the quasi-static ones provide valid results typically below 5 GHz [1]. If the method is required to provide parameters of an arbitrary structure, the use of an analytical model [4] cannot be taken into consideration because an analytical effort is required for every structure. Taking remaining methods in mind, the finite-element method is preferred from the following reasons [14]:

a) The computed quantity has a uniquely defined value everywhere within the analysed area (not only in nodes as in the finite-difference method).

b) The approximate solution has minimal error in a global sense that takes into account the solution values at all points (not only the nodal ones as the finite-difference method).

c) There are no limitations to the shape and to the size of elements. Therefore, the elements can be curvilinear to match geometry of the analysed structure, they can be small in areas where the dramatic changes of a field are supposed and large where a slowly varying field is expected. This is not possible if the finite-difference method is used.

Therefore, the finite-element method has been chosen for the analysis of general micro- and millimeter-wave transmission lines.

In the first chapter of this work, an overview of vectorial hybrid nodal-edge finite elements [6] is given and some aspects of the already existing method are originally discussed.

Since accuracy of results obtained by the finite-element method is crucially influenced by the quality of the finite-element mesh [13] [14], this influence is discussed in the second chapter. Accuracy of obtained results is tested using the reaction concept and taking analytical models as the reference.

If an optimal finite-element mesh is at the disposal, the finite-element matrix equation can be built and it has to be efficiently solved. The efficient solution used in this work comes from the finite-element complex-hopping method [5] which is originally extended to the analysis of transversally non-homogenous structures in the chapter 3.

All the computations described in chapters 2 and 3 are performed for shielded waveguides because the finite-element method can be used for the analysis of closed systems only [13] [14]. If an open waveguiding structure is going to be analysed then the open system has to be converted to the closed one. In chapter 4, a special real spatial mapping [15] is described and it is originally implemented to the analysis of open waveguides. Moreover, the introduced mapping can be shown to behave as a perfectly matched layer [17] which is shown at the end of the chapter 4.

Developed methods are useful not only from the technical point of view but from the pedagogical one too because students can model various structures using these methods. Fields can be visualised, frequency dependencies of propagation parameters can be plotted which all helps to students in understanding complex wave phenomena in transmission lines. This topic is discussed in the habilitation but it was to missed here because of the shortage of place.

1 Finite-element analysis of transversally non-homogenous micro- and millimeter-wave transmission lines

Finite element method (FEM) is the general approach to the solution of partial differential equations. Since Maxwell equations expressed in terms of differential operators belong to the above class of equations, FEM is widely used to solve them [13] [14].

Applying FEM to the analysis of transversally non-homogenous waveguides is rather difficult because classical versions of FEM produce spurious (physically non-existing) solutions [6] [14]. The first source of spurious solutions is related to the boundary conditions at the interface between dielectric layers (some of the tangential boundary conditions, which are necessary to unambiguously define the boundary value problem, are not satisfied) and the second one to the presence of sharp metallic edges (electric field approaches infinity at the edge and field's direction changes infinitely there) [6].

1.1 Vectorial basis functions

As a solution of the described problems, tangential vector finite elements (TVFE) were proposed [6]. TVFE impose only tangential continuity of the field, and therefore, the modelled electromagnetic (EM) field can change abruptly at sharp edges [6].

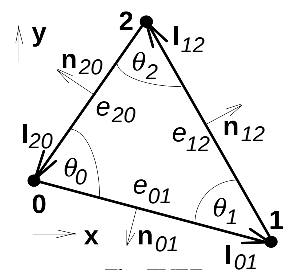

Fig. 1.1 *The TVFE*

The occurrence of the spurious modes due to the unsatisfied tangential boundary conditions can be interpreted as the improper modelling of the null space of the curl operator. If appropriate TVFE are used, null space of the curl operator is modelled exactly and the spurious modes degenerate to eigensolutions with eigenvalue zero [6].

The main idea of TVFEs consists in approximating transversal components of the electric-intensity vector by edge finite elements (FE) and in approximating longitudinal components by nodal ones.

1.2 Functional and its minimization

Assume a longitudinally homogenous waveguiding structure closed by perfect-electric conductor (PEC) or perfect-magnetic conductor (PMC) walls. The structure can consist of a linear dielectric filling of arbitrary properties (non-homogenous, anisotropic, lossy) and of arbitrary shape. Arbitrary metallic objects can be present in the structure.

The described structure is placed into the Cartesian coordinate system so as the cross section of the waveguide can lie in the *x, y* plane and the longitudinal axis of the waveguide can be of the same direction as *z* axis of the coordinate system. Then, the *z*-dependence of the EM field inside the structure can be expressed as $\exp(-\gamma z)$ where γ is the complex propagation constant [1].

The so far published FE methods for the analysis of structures meeting the above description have been based on the Galerkin's method [6] [14]. In our original approach, the analysis comes from a functional, which was published by I. Huynen and A. Vander Vorst [4], and which is stationary about the solution of the vectorial wave equation describing the EM field in the structure.

The functional was re-formulated in terms of the transversal electric-intensity vector and the longitudinal electric-intensity component. Performing some mathematical manipulations and enforcing the relations to fulfil Dirichlet and Neumann boundary conditions on PEC and PMC walls, the Euler equation associated with the functional was obtained. Manipulating the Euler equation, the divergence-free condition for the non-zero frequency was obtained. Therefore, the introduced mathematical formulation ensures that spurious solutions, which could be produced due to the uncomplete fulfilling boundary conditions, can never appear.

Substituting the FE approximations to the functional, introducing the necessary boundary conditions, extremizing the functional and summing resultant equations over all the elements yield the matrix relation

$$\left[\gamma^2\mathbf{M}_1+\gamma\,\mathbf{M}_2+\mathbf{M}_3\right]\boldsymbol{E}=\mathbf{0} \qquad (1.1)$$

In the above equation, γ denotes the complex propagation constant, $\boldsymbol{E}$ is the column vector of unknown coefficients of the FE approximation, $\mathbf{0}$ denotes the column vector of zeros and $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$ are matrices containing products of shape functions and their derivatives integrated over FEs.

If the operator in a vectorial wave equation associated with the functional is self-adjoint then the resultant matrix equation (1.1) is identical with the result of Galerkin's method [14]. Otherwise, the resultant matrix equation is identical with the result of Galerkin's method if and only if the adjoint trial fields in the functional are expanded by non-adjoint basis functions [14]. Since the above presented basis functions are the same both in the adjoint and in the non-adjoint form, the matrix equation (1.1) should be identical with the result obtained on the basis of Galerkin's method. And it really is.

If the operator in a vectorial wave equation associated with the functional is non-negative definite then the FE approximation converges to the extreme of the functional in the sense of energy [14]. Otherwise, the FE approximation exhibits weak convergence [14].

1.3 Solution of the matrix equation

If propagation constants γ and discrete values of the field components are required, then the matrix equation (1.1) has to be solved. The most usual way of doing this is transforming (1.1) into the classical generalized eigenvalue problem [6]

$$\left\{\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{M}_3 & \mathbf{M}_2\end{bmatrix}-\gamma\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}_1\end{bmatrix}\right\}\cdot\begin{bmatrix}\boldsymbol{E}\\ \tilde{\boldsymbol{E}}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\end{bmatrix} \qquad (1.2)$$

which can be efficiently solved by the classical eigenvalue solvers such as the `eig` function of matlab.

In (1.2), **I** denotes unitary matrices of respective size, $\widetilde{\boldsymbol{E}} = \gamma\,\boldsymbol{E}$ and the rest of symbols has the same meaning as it was described below eqn. 1.1.

1.4 Conclusions

In this chapter, the FEM for the full-wave analysis of shielded longitudinally homogenous microwave structures is described. Analysed structures can consist of a non-homogenous dielectric exhibiting anisotropy and losses and can contain arbitrarily shaped lossy metallic parts.

The analysis comes from the functional, which is stationary about the solution of a vectorial wave equation describing the structure. A solution of the functional is approximated in terms of vectorial nodal-edge FEs. Then, the functional is minimized, which produces quadratic eigenvalue equation for complex propagation constants as eigenvalues and approximation coefficients of the field distribution as eigenvectors. The quadratic eigenvalue equation is converted into the linear one which is solved by the generalized eigenvalue solver of matlab (function `eig`).

Dealing with the CPU demands, the computations require about 10 seconds per frequency when Matlab 5.1 running under Digital UNIX 4.0c on the Personal Workstation Digital 433au is used.

The original work of the first chapter consist in the use of the variational expression of Huynen and Vander Vorst as the initial equation of the analysis and in comparison of the variational approach with the Galerkin's method.

2 Quality of the finite-element mesh

Accuracy of obtained results is a very important question which has to be answered with respect to FE solutions. For structures, which can be analysed by some different method or the parameters of which can be measured, comparison of results and computing the relative error of the FE solution can be done easily. For structures, which have not been so far analysed by any different method or parameters of which have not been measured so far, the method based on the reaction concept has been worked up [6].

In the first section of this chapter, the reaction concept is reviewed and results of the FE analysis of the shielded microstrip line, obtained with various FE meshes, are compared with results of the analysis based on the analytic model. Moreover, it is pointed out that there is a coincidence between the errors based on the reaction concept and on the analytic model.

In the last section, possible ways of the automated FE mesh optimization are discussed. Then, a simple FE mesh refinement based on the Non-Linear Random Search (NLRS) algorithm is proposed. The structure of the NLRS algorithm is compared with structures of artificial neural networks and it is pointed out that there are common features of both the systems.

2.1 The reaction concept

Quality of the FE mesh with respect to the accuracy of the FE solution can be evaluated according to the procedure based on the reaction concept. This procedure comes from the definition of the reaction of a field a on a source b and from the consideration that the true field at resonance is source-free. Therefore, the reaction of any field with the true source is zero [6].

Taking the above conclusion in mind, two separate analyses of the structure, which come from dual variational expressions, are performed, sources are computed from Maxwell's equations and the relative error provided by eth element is evaluated according to the relation [6]

$$\delta_e = \frac{\left\| \int_{\Omega} \left(\mathbf{E} \cdot \mathbf{J} - \mathbf{H} \cdot \mathbf{M} \right) d\Omega \right\|}{\omega \int \left(\frac{1}{2} \varepsilon \|\mathbf{E}\|^2 + \frac{1}{2} \mu \|\mathbf{H}\|^2 \right) d\Omega} \qquad (2.1)$$

with electric- and magnetic-intensity vectors **E** and **H**, with electric and magnetic current-density vectors **J** and **M**, with permittivity and permeability within a FE ε and μ, with angular frequency ω and with integration over a FE.

The global error can be computed then as the sum of δ_e over all the FEs.

Performing FE analysis for various FE meshes and evaluating the relative error using both the reaction concept and the analytic model as a reference value, following conclusions were done:

1. The good accuracy of obtained results is conditioned by the *good* homogenity of the FE mesh which means that all the FEs have to be of approximately the same size.

2. There is a coincidence between the relative errors computed by the reaction concept and by the comparison with the analytic model results. On the other hand, the FEM provides results which does not converge to the analytic model ones - there is a certain shift between results obtained by both the methods.

2.2 Mesh optimization based on Non-Linear Random Search Algorithm

A Linear Random Search (LRS) algorithm was proposed to enable optimization of system with an unknown mathematical model. The algorithm can be described by the relation [16]

$$\mathbf{W}_{n+1} = \mathbf{W}_n + \beta\left[\tilde{\xi}(\mathbf{W}_n) - \tilde{\xi}(\mathbf{W}_n + \mathbf{U}_n)\right]\mathbf{U}_n \qquad (2.2)$$

where **W** is the column vector of state variables, **U** is the column vector of random numbers having the covariance σ^2 **I** (**I** is the unitary matrix), $\tilde{\xi}(\mathbf{W}_n)$ denotes a squared error (difference between the solution related to the state vector $\mathbf{W}_n$ and the exact one) and β and σ are adaptation parameters of the algorithm.

In the algorithm, a state vector $\mathbf{W}_n$ is introduced to the optimized system and the squared error is measured. In the next step, a vector of random numbers is added to the state vector, this addition is introduced to the optimized system and the squared error is again measured. If the random change of the state vector $\mathbf{W}_n + \mathbf{U}_n$ causes a decrease of the squared error then the state vector is changed in the same direction in which the random change of the state vector has been performed. In the opposite case, the state vector is changed in the contra-direction of the random change.

It is obvious that the exact mathematical model of the optimized system is not needed in the LRS algorithm, and therefore, this algorithm can be used for the optimization of the FE mesh with respect to the squared error of the finite-element solution.

Applying the algorithm to the FE mesh optimization, sizes of FEs in the direction **x**: $dx(1)$, $dx(2)$, ..., $dx(N_x)$ and sizes of FEs in the direction **y**: $dy(1)$, $dy(2)$, ..., $dy(N_y)$ are considered to form the state vector **W**. The squared difference of the complex propagation constants of the dominant mode of solutions performed in electric-intensity terms and magnetic-intensity ones is taken as the squared error (* is complex conjunction)

$$\tilde{\xi}(\mathbf{W}_{n+1}) = \left[\gamma_E(\mathbf{W}_n) - \gamma_H(\mathbf{W}_n)\right]\cdot\left[\gamma_E(\mathbf{W}_n) - \gamma_H(\mathbf{W}_n)\right]^* \qquad (2.3)$$

Moreover, a non-linearity, which limits random changes of the size of FEs so as they can correspond to the size of the analysed structure, has to be introduced to the *so-far-linear* algorithm.

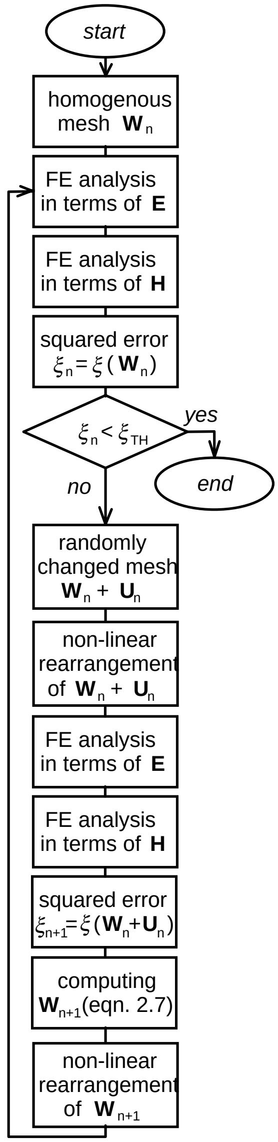

Fig. 2.1 *Block scheme of the NLRS algorithm*

The block scheme of the described algorithm is depicted in fig. 2.1. In this scheme, the squared error ξ_{TH} denotes the threshold squared error. If the FE mesh produces lower squared error than ξ_{TH}, then the optimization is stopped.

At the end, let's try to classify the developed NLRS algorithm as it were an artificial neural network. In our situation, the classical non-linear model of neuron is modified by moving the non-linear activation function from the output of summer into its input branches and by the formal introduction of activation functions into weights. The modified non-linear neuron implements one row of the final FE matrix equation (1.2). Input signals correspond to coefficients of edge or nodal FEs, synaptic weights represent sizes of FEs in directions **x** and **y**, activation functions and thresholds submit to weights' modification within the non-linear random search algorithm, linear functions build coefficients of matrices from (1.2) on the basis of FE sizes and simplex matrices and summer computes right-hand side of (1.2) as the function of unknown complex propagation constant γ. In the modified neuron, thresholds are random numbers generated by the standard matlab function `rand`. These thresholds correspond to the random vector **U** from eqn. 2.2.

On the basis of the above description, the FE routine completed by the NLRS mesh optimization can be considered to be built from a special kind of neurons - non-linear modified ones [2].

From the point of view of learning paradigms, the NLRS neural network exhibits supervised learning. If the analytic-model results are taken as the learning pattern, the analytic model play the role of the teacher. If the optimization process is based on the reaction concept then the error signal, which is computed by the comparison of the teacher's results and the optimized system ones, is built as a difference of solutions based on electric-intensity components and magnetic-intensity ones [2].

Dealing with the architecture of the neural network, the developed network can be considered as a single-layer feed-forward network with the back propagation of the error [2].

Therefore, the final conclusion can state that the NLRS algorithm for the optimization of the FE mesh can be considered as an artificial neural network [2].

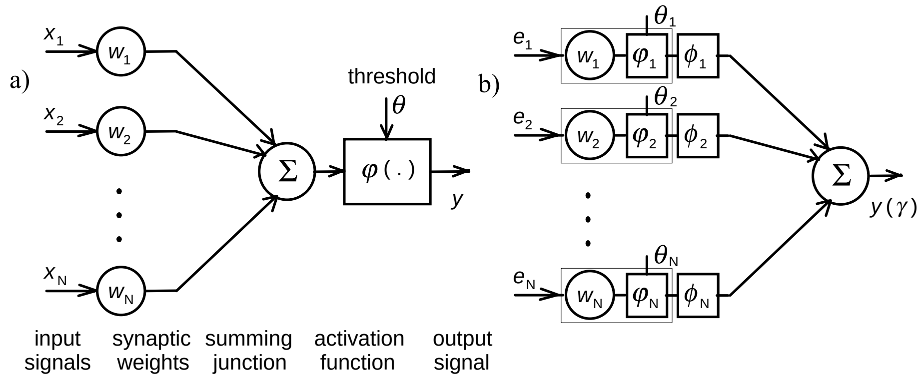

Fig. 2.2 *Non-linear models of neuron: a) classical, b) modified*

2.3 Conclusions

This chapter reviews methods which enable evaluation of the quality of FE meshes. Using these methods, original discussions and comparisons of various FE meshes have been done. The meshes have been shown to have to be *well* homogenous to provide relatively accurate results.

Moreover, the correspondence between errors computed from the reaction concept and from the analytic model has been originally discussed and both errors have been shown to exhibit a certain shift between each other.

On the basis of obtained results, an original algorithm for the FE mesh optimization has been developed. The algorithm comes from the Linear Random Search algorithm, which does not require mathematical description of the optimized system. The algorithm has been completed by the non-linear activation function which limits sizes of FEs so as they cannot be too small or too large and which matches sizes of finite elements to the geometry of the analysed structure.

The developed NLRS algorithm has been shown to be able being considered as an artificial neural network consisting of modified non-linear neurons, exhibiting supervised learning and having the single-layered feed-forward architecture with back propagating error signal.

3 Finite-element complex-hopping method

Computational effectiveness of the FE analysis of waveguiding structures is crucially influenced by the solution of linear or quadratic eigenvalue problem produced by the method [14]. To overcome this difficulty, M. A. Kolbehdari applied complex frequency hopping (CFH) technique to the computation of critical frequencies of the transversally homogenous rectangular waveguide and have shown that speed-up ratios of CFH in comparison with eigensolution based on LU decomposition are from 19 to 72 depending on the matrix size [5]. Even more, CFH exhibits good accuracy and does not require development of any special mathematical routine (CFH can be very effectively implemented by sparse matrix operations from the matlab core).

Thanks to the high speed, good accuracy and simple programming, CFH seems to be attractive for the use in the analysis of general waveguiding structures. Therefore, the CFH concept is originally applied to the analysis of transversally non-homogenous waveguiding structures. Starting at the vectorial wave equation, a functional is derived which can be shown to exhibit variational behaviour, and, under given circumstances, comes to the new variational formula published by I. Huynen and A. Vander Vorst [4]. The functional is then solved by hybrid nodal-edge FEs [6] and complex hops (CH).

3.1 Complex propagation constant hopping

Assume a cylindrical waveguide of an arbitrary cross section which is bounded by PEC or PMC walls, which is filled by linear lossy non-homogenous anisotropic dielectrics described by the permittivity, permeability and electric conductivity tensors and which contains arbitrary metallic parts. If a harmonic wave is assumed then the vectorial wave equation can be expressed in the form

$$\nabla \times \left(\overline{\mu}^{-1} \cdot \nabla \times \boldsymbol{E}\right) + s^2 \overline{\varepsilon} \cdot \boldsymbol{E} - s\overline{\sigma} \cdot \boldsymbol{E} = s\boldsymbol{J} \qquad (3.1)$$

Here, **E** is the electric intensity vector, **J** denotes the source current density vector, s is the angular complex frequency, ε, μ and σ are permittivity, permeability and electric conductivity tensors respectively.

Vectors in (3.1) are split into transversal and longitudinal components. Then, the relation is multiplied by the complex conjugate electric intensity vector and is integrated over the waveguide cross section. Applying Green's theorem and considering the boundary conditions yield an original functional which can be shown to be variational.

Now, the waveguide cross section is divided into triangular FEs, transversal electric field components are approximated in terms of vectorial edge FEs and longitudinal one is approximated in terms of nodal FEs [6]. Approximations are substituted into functional which is then minimised with respect to the z-dependent approximation coefficients. Applying Laplace transform to the produced matrix equation with respect to the longitudinal component z yields

$$\mathbf{Y}(\gamma) = \mathbf{H}(\gamma, s)\mathbf{X}(\gamma) \qquad (3.2)$$

with the column vectors of input and output spatial spectra **X** and **Y** respectively and with the transfer function of the analysed structure **H**.

The output signals' spatial spectra **Y** can be expanded by the Taylor's series about the complex propagation constant γ_0 and the transfer function between the excitation point and the output one $H_{out,exc}(\gamma,s)$ can be approximated by the fractional function

$$H_{out,exc}(\gamma,s) = \frac{\sum_{l=0}^{L} a_l(s)\gamma^l}{1+\sum_{k=1}^{K} b_k(s)\gamma^k} \qquad (3.3)$$

with the complex propagation constant γ and with the complex frequency s.

If the Taylor's expansion of output spatial spectra

$$\mathbf{Y}(\gamma) = \sum_{n=0}^{\infty} \mathbf{M}_n (\gamma - \gamma_0)^n \qquad (3.4)$$

and the fractional function (3.3) are substituted into (3.2), if a white noise excitation in the sense of spatial spectra is assumed and if coefficients at the same powers of γ are compared then relations for computing coefficients of the fractional function are obtained.

The derived relations can be now used for the complex propagation constant hopping which provides approximation of the transfer function in the interval of interest $<\gamma_{min}, \gamma_{max}>$ for the given complex frequency s. Propagation constants, by which wave can propagate on the given frequency along the analysed structure, are determined by poles of $H_{out,exc}$.

3.2 Conclusions

The original application of the CH to the FE analysis of shielded waveguides has been described. The analysis come from an originally derived functional which exhibits variational behaviour for trial fields fulfilling Maxwell's equations. The functional is identical with the new variational formula of I. Huynen and A. Vander Vorst when given conditions are met.

Functionals are solved in terms of hybrid nodal-edge FEs and CH is applied to the resultant matrix equation. The technique is based on the expansion of the kernel of the eigenvalue equation to the set of low-order fractional functions. Then, eigenvalues can be easily found by computing poles of low-order expansion functions.

Obtained results shows good accuracy, simple programming and low CPU costs.

4 Finite-element analysis of open structures

The FEM cannot be directly used for the analysis of open waveguiding structures because the analysed area has to be bounded by the surface of known boundary conditions [13] [14]. Therefore, if an analysis of an open structure is required then the FEM has to be completed by an additional technique which converts the open problem to a closed one. At the present time, perfectly matched layers (PML) are most frequently used [17].

PML can be understood as an artificial lossy material which efficiently attenuates waves falling to it and which exhibits no reflections for all frequencies, angles of incidence and polarizations [17].

In this chapter, a real spatial mapping, which transforms an infinite space into a finite one [15], is introduced into Maxwell's equations. By this way, a layer, which surrounds an open longitudinally homogenous waveguide and which efficiently attenuates transversally propagating evanescent waves, is created. Moreover, the introduced spatial mapping is interpreted as a PML based on the real stretch of coordinates [17] and on a lossles anisotropic layer.

4.1 A real spatial mapping interpreted as a PML

Assume that electromagnetic field of an open isotropic waveguide is going to be analysed. The infinite space surrounding the structure is divided into an inner region, in which the detail information about the EM field is required, and an outer one, which is mapped into a layer of finite thickness [15]

$$m' = a_m R_m - \frac{(a_m - 1)R_m^2}{m} \qquad a_m > 1 \qquad m = x, y, z \tag{4.1}$$

The mapping (4.1) is invariant to the border between the inner and outer region R_m and the infinity is "shifted" to the distance $a_m R_m$. Therefore, the area of interest can be imagined to be surrounded by a layer of the relative thickness a_m which replaces the original infinite space. Since the evanescent waves are supposed to be zero in the infinity, the external surface of the layer can be covered by PEC or PMC walls.

If electric and magnetic intensity vectors are expressed in terms of primed coordinates then Maxwell's equations have to be rewritten using the modified del operator

$$\nabla_a = \mathbf{x}\frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \mathbf{y}\frac{\partial y'}{\partial y}\frac{\partial}{\partial y'} + \mathbf{z}\frac{\partial z'}{\partial z}\frac{\partial}{\partial z'} = \mathbf{x}\frac{1}{s_x}\frac{\partial}{\partial x'} + \mathbf{y}\frac{1}{s_y}\frac{\partial}{\partial y'} + \mathbf{z}\frac{1}{s_z}\frac{\partial}{\partial z'} \tag{4.2}$$

where

$$s_m = \frac{(a_m - 1)R_m^2}{(a_m R_m - m')^2} \qquad \text{with} \qquad m = x, y, z \tag{4.3}$$

are obtained from (4.1). If a primed coordinate approaches the border between the inner and outer region R_m then s_m is a real number depending on the relative thickness of the surrounding layer a_m. If a primed coordinate approaches the external surface of the layer

$a_m R_m$ then s_m converges to infinity. If an evanescent plane wave shall be attenuated in direction m' then the condition $s_n = 1$ for all $n \neq m$ has to be fulfilled to guarantee no reflections at the interface R_m. Since $n = n'$ and $\partial n'/\partial n = 1$ if no mapping is applied to directions $n \neq m'$, the above condition is fulfilled and no reflections appear at the internal surface of PML. The amplitude decay at the m' direction is then described by the following relation

$$\psi = \psi_0 \exp\left(-k_m'' s_m m'\right) \qquad \text{with} \qquad \psi = \mathbf{E}', \mathbf{H}' \tag{4.4}$$

Here, k_m'' is the attenuation constant of the evanescent wave falling to the PML in m' direction.

It is obvious (4.4) that the attenuation of the evanescent wave is increased by the factor s_n in the PML. The factor s_n equals $1/(a_n$-$1)$ at the inner border of PML and continuously grows to infinity at the external surface of PML which models well the PEC or PMC coverage of the layer.

Following the intellectual line of [17], transformed space can be considered as an anisotropic layer described by space-dependent permittivity and permeability tensors and with physical fields replaced by scaled ones.

4.2 Conclusions

The original perfectly matched layer based on the real spatial mapping is described in this chapter. The PML is shown to efficiently attenuate evanescent waves, and therefore, it can be adopted for the FE analysis of open microwave transmission lines.

The presented PML exhibits high attenuation ability. That is why the space surrounding the open waveguide can be minimized, and therefore, the FE analysis of the structure shows high efficiency.

Functionality of the proposed method has been verified by computing complex propagation constants of the dominant mode of the open microstrip line and open image line. Obtained results have exhibited good agreement with other methods. Among results, no spurious solutions have been observed.

5 Conclusion

In the framework of the presented theses, the so far development of FE techniques for the analysis of general, longitudinally homogenous microwave transmission lines have been reviewed and originally extended. The analysed structures could be both shielded and open, both lossy and lossles, they could exhibit arbitrary non-homogenity and anisotropy.

The developed FEM for the analysis of shielded structures has come from the functional derived by I. Huynen and A. Vander Vorst. In this functional, trial fields were approximated in terms of vectorial hybrid nodal-edge FEs and the functional was minimized with respect to approximation coefficients. The resultant matrix equation has been shown to equal the Galerkin's solution.

Since accuracy of the FE solution is significantly influenced by the FE mesh covering the analysed structure, the detailed discussion has been devoted to meshes. Many various meshes have been proposed and their quality has been tested by the method based on the reaction concept and by comparing FE results with analytic-model ones. Then, the FE mesh optimization procedure based on the non-linear random search algorithm has been proposed and it has been shown to behave as an artificial neural network. Thanks to the introduced mesh optimization, the relative error of the FE solution was reduced to the level of few per cent.

Solution of the generalised eigen-value problem is the most time-consuming part of the FEM, and therefore, the eigen-problem has been replaced by the CH procedure. Thanks to the described replacement, computational requirements of the method have been significantly reduced.

The developed CH techniques have come from an original functional which has been minimized in terms of nodal-edge FEs. Minimizations of the functional has yielded a matrix equation which has presented a waveguide as a linear system with a given transfer function. Observing resonances of the transfer function has revealed complex propagation constants which could appear on a given frequency.

Since the classical FEM can be applied to the analysis of shielded structures only, a special attention has been paid to the analysis of open waveguides using a special spatial mapping. The mapping has been shown to behave as a PML surrounding the structure.

The use of the developed algorithms in the teaching process seems to be the other very interesting topic which should be discussed. This discussion was done in the habilitation and it was missed here because of the shortage of place.

The presented work has tried to cover all the aspects of the FE solution of micro- and millimeter-wave transmission lines, to review them and to extend them by an original way.

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Obsahem habilitační práce „*Full-Wave Finite-Element Analysis of General Microwave Waveguides*“ bylo vytvořit kritický přehled dosud vyvinutých numerických metod, založených na konečných prvcích, a sloužících pro analýzu podélně homogenních mikrovlnných vedení. Na základě zjištěných nedostatků byly tyto metody originálně rozvinuty. Důraz byl přitom kladen na obecnost vyvíjených postupů - algoritmy měly umožňovat analýzu stíněných i otevřených, ztrátových i bezeztrátových, isotropních i neisotropních lineárních struktur.

Základní metoda vychází z funkcionálu I. Huynenové a A. Vander Vorsta, který popisuje rozložení elektromagnetického pole v mikrovlnném vedení. Složky pole byly v tomto funkcionálu aproximovány pomocí hybridních hranově-uzlových konečných prvků a funkcionál byl minimalizován vzhledem k neznámým aproximačním koeficientům. Byla tak získána maticová rovnice, jejíž tvar byl shodný s rovnicí, produkovanou Galerkinovou metodou.

Jelikož přesnost metody konečných prvků je výrazně ovlivněna kvalitou použité sítě konečných prvků, byla detailní diskuse věnována i této otázce. Bylo navrženo několik sítí a jejich kvalita byla testována pomocí konceptu reakce a pomocí srovnání výsledků konečných prvků s výsledky, produkovanými metodou analytického modelu. Poté byl navržen algoritmus optimalizace sítě, založený na nelineárním náhodném hledání, a navíc bylo ukázáno, že lze tento algoritmus považovat za umělou neuronovou síť. Díky zavedené optimalizaci byla minimalizována chyba, produkovaná metodou.

Jelikož nejvíce výpočetního času spotřebovává u metody konečných prvků řešení zobecněného problému vlastních čísel, byla věnována detailní pozornost i této otázce. Problém byl řešen hybridizací metody konečných prvků s metodou komplexního přeskakování. Tím byla nahrazena nutnost řešit zobecněný problém vlastních čísel výpočtem soustavy lineárních rovnic, jenž spotřebovává mnohem méně času. Přesnost metody je pouze nepatrně nižší nežli je tomu u klasického přístupu. Cenou, kterou musíme za větší rychlost výpočtu zaplatit, je nemožnost získat informaci o rozložení pole (metoda produkuje pouze komplexní konstanty šíření vidů, jež mohou ve struktuře vzniknout).

Jelikož klasická metoda konečných prvků může být aplikována pouze na stíněná mikrovlnná vedení, byla pozornost věnována i otázce možné analýzy otevřených struktur. Na otevřené vedení byla aplikována speciální prostorová transformace, u níž bylo dokázáno, že se chová jako dokonale přizpůsobená vrstva, a která výrazně zefektivnila proces analýzy díky přesunutí nekonečna, v němž je pole nulové, do konečné vzdálenosti od analyzované struktury.

Lákavým tématem k diskusi je rovněž možnost použití vyvinutých algoritmů ve výuce. Této otázce byla věnována pozornost v habilitační práci, avšak v těchto tezích nebyla vzhledem k nedostatku místa tato otázka diskutována.