BRNO UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering and Communication

Doc. Ing. Roman Prokop, CSc.

ALGEBRAIC METHODS IN DESIGN AND ANALYSIS OF ROBUST CONTROLLERS

ALGEBRAICKÉ METODY V NÁVRHU A LADĚNÍ REGULÁTORŮ

TEZE PŘEDNÁŠKY KE JMENOVÁNÍ PROFESOREM
V OBORU TECHNICKÁ KYBERNETIKA

VUTIUM

BRNO 2004

Key words: ring of rational functions, robustness, asymptotic tracking, disturbance rejection, diophantine equations

Klíčová slova: okruh racionálních funkcí, robustnost, asymptotické sledování, potlačení poruch, diofantické rovnice

Abstract: The contribution is focused on the design and tuning of simple continuous-time SISO regulators of PID-like structures. All desirable controllers are obtained in the parameter form of general solutions of linear equations in the ring of proper and Hurwitz-stable rational functions. The final controller is then chosen for given specifications, e. g. robustness, asymptotic tracking, disturbance rejection, ... A scalar parameter is introduced for tuning of final derived controller parameters. Perturbations and uncertainties are studied through the H_∞ norm, Kharitonov´s and zero exclusion theorem.

Abstrakt: Práce se zabývá návrhem a laděním spojitých regulátorů se strukturou PID. Všechny stabilizující regulátory se získají v parametrickém tvaru jako obecné řešení lineární diofantické rovnice v okruhu ryzích a (Hurwitz) stabilních racionálních funkcí. Výsledný regulátor se pak vybírá podle dalších specifikací jako je robustnost, asymptotické sledování, potlačení poruch a pod. Pro ladění a ovlivňování regulačních pochodů byl navržen skalární kladný parametr, kterého funkcí jsou jednotlivé koeficienty regulátoru. Pro studium perturbací, neurčitostí a robustnosti se používá aparát normy H_∞, Charitonovovy věty a podmínky vyloučení nuly.

©2003 Roman Prokop
ISBN 80-214-2537-7
ISSN 1213-418X

AUTHOR'S INTRODUCTION

Name: Roman Prokop, (Doc., Ing., CSc.)
Date of birth: 15.10. 1952

Education and degrees:
1971-1976 Ing.(MSc) - Faculty of Nuclear Science and Physical Engineering, CTU Prague
1978-1983 CSc.(PhD) - Slovak Technical University, Faculty of Electrical Engineering
1996 Doc. (Assoc.prof.) - Technical University Brno, Faculty of Technology in Zlín

Professional experience:
1976 – 1995 Slovak Technical University, Faculty of Chemical Technology in Bratislava, Department of Automatic Control, assistant.
1995 – 2000 Technical University of Brno, Faculty of Technology in Zlin, Process Control Department, vice dean, associate professor.
Tomas Bata University in Zlin, Faculty of Technology, Institute of Information Technologies, vice dean, associate professor.

Research activities:
Algebraic methods in control design (discrete and continuous-time, SISO and MIMO systems),
Recursive identification methods, adaptive self-tuning controllers, MRAS,
Robust control and uncertainty, time-delay systems, time-varying systems.

Co-operation and mobility:
Kovohute, Břidličná - aluminum alloy process control, 1986-92
University of Birmingham, U.K., 6 months, TEMPUS JEP,1992-93
Uppsala University, Uppsala, Sweden, 1994
ENSIC - INPL Nancy, France, 3 months, 1995, 1997
University of Strathclyde, Glasgow, U.K., 1997
LAAS CNRS Toulouse, France, 1999
INCO Copernicus CP 97 - Europoly, 1998-2001

Publications:
Author or co-author 2 monographs, 5 study texts
Author or co-author 37 journal publications (19 internationals)
Author or co-author 42 presentations on the scientific international conferences
Author or co-author 68 presentations on the scientific national conferences

CONTENTS

1 INTRODUCTION

The achievement of simple but robust, reliable but insensitivity controllers is a fundamental and never ending task in control theory. Algebra has received more and more attention and gained reputation in system analysis and design during last decades (e.g. Doyle at al., 1992; Kučera, 1979, 1990; Vidyasagar, 1985,). Algebraic notions such as rings, domains, ideals, linear equations (Diophantine or Bezout) seem to be powerful and effective tools for the design in linear systems as well as in signal processing, 2-D systems, and so on.

In this framework, this paper concerns the methodology of how to obtain simple SISO controllers from any particular solution of diophantine equations solving the basic stability problem. Resulting controllers are expected to have various useful properties. Usually, stability and asymptotic tracking is a natural requirement. Another specifications of controllers can be formulated through the properness, the ability of the disturbance rejection, robustness to parameter uncertainty.

According to the basic idea adopted from Vidyasagar (1985), Kučera (1993), a system transfer function is expressed as a ratio of two stable rational functions. All stabilizing controllers are then given by parameter solutions of a diophantine equation in the appropriate ring. Other specifications are then formulated as divisibility conditions in the ring and are equivalent either to other diophantine equation or to a special choice from the set of all solutions.

It is also shown that manipulation with polynomials does not offer the possibility of the sequential choice of proper controllers and that the same problem would be solved in the ring of polynomials more awkwardly and clumsily. The approach is illustrated with simple examples which confirm the modified PID controllers proposed by Åström, 1991.

2 SYSTEM DESCRIPTION OVER RINGS

Continuous-time (CT) linear systems have traditionally been described by polynomial fractions in the derivative operator s. A controlled plant is supposed to be a linear, time-invariant dynamic system modelled as a rational transfer function whose input u and output y are scalar quantities in a case of SISO systems. The fact that the set of polynomials is a ring is well known. However, other rings exist which could be used in control design.

Roughly speaking, a commutative ring is a (non empty) set equipped with two operations, addition and multiplication (both commutative). If the ring has an identity, then its elements having a multiplicative inverse are called units of the ring. A ring in which every non-zero element is a unit is called a field. More details about properties of various rings can be found in standard textbooks of algebra or in Vidyasagar (1985), Kučera (1979). Note, that a set of all rational functions in the indeterminate s (of complex plane) is a field. Various subsets of this field create rings (see Kučera, 1993). It can be easily verified that a set of polynomials $\mathbf{P}$ (analytic functions for every $s \neq \infty$) as well as the set $\mathbf{R}_{PS}$ of Hurwitz stable and proper rational functions are rings. The set $\mathbf{R}_{PS}$ contains all rational fractions which have no pole in $Re(s) \geq 0$ including ∞ (analytic in the extended right complex half-plane). Units in the ring **P** are non zero constants, while the units in $\mathbf{R}_{PS}$ are miniphase functions (no zeros in Re(s) ≥ 0 including ∞) of the relative degree zero. Two elements in **P** are coprime if they have no common roots while two elements of $\mathbf{R}_{PS}$ are coprime if they have no common zeros in Re(s) ≥ 0.In some applications of continuous-time systems, there is another ring of relevant importance $\mathbf{D}_{PS}$. Thus $\mathbf{D}_{PS} \subset \mathbf{R}_{PS}$ is a set of rational functions having poles only in a pre-specified subset of the left half-plane. The frequent case of such region is D={ s: Re(s) <-m_0; m_0 > 0}. It can be easily showed (Vidyasagar, 1985) that $\mathbf{D}_{PS}$ is also a ring. Two elements of $\mathbf{D}_{PS}$ are coprime if they have no common zeros outside D. It is clear that $\mathbf{D}_{PS} = \mathbf{R}_{PS}$ when D is the whole left half-plane. In all successive formulations $\mathbf{R}_{PS}$ can be replaced by $\mathbf{D}_{PS}$ when appropriate.

Then a transfer function of a CT linear causal system can be expressed in two equivalent forms:

$$H(s) = \frac{B(s)}{A(s)} = \frac{a(s)}{b(s)}; \quad A(s) = \frac{a(s)}{m(s)}; \quad B(s) = \frac{b(s)}{m(s)} \tag{1}$$

where A,B∈$\mathbf{R}_{PS}$ (or generally in $\mathcal{D}$) ; a,b∈ **P** and m is a stable polynomial with deg m = max{deg a, deg b}. The H_∞ norm of an element of $\mathbf{R}_{PS}$ suitable for uncertainty and robustness is defined as

$$\|H\| = \sup_{\mathrm{Re}\, s \geq 0} |H(s)| = \sup_{\omega \in E} |H(j\omega)| \tag{2}$$

where the second quantity is due to the maximum modulus theorem and E is a set of real numbers. This norm is the radius of the smallest (centred at origin) circle containing the Nyquist plot of the transfer function. Since almost all mathematical models differ from physical systems, a control designer should possess tools enabling to express the influence of modelling errors on the performance of a control system. The notion of robustness through norms constitutes such a tool. Let *H* be a nominal plant given by (1), then we consider a family of perturbed systems with respect to (1) consisting of all transfer *functions H' = B'/A'*, where *A', B'* are elements of $\mathbf{R}_{PS}$ and

$$\|A - A'\| \leq \varepsilon_1, \qquad \|B - B'\| \leq \varepsilon_2 \tag{3}$$

3 FEEDBACK SYSTEMS AND STABILITY

A typical control problem can be formulated as follows: Given a plant (by its transfer function), find a controller (or family of controllers) such that the feedback system is stable (in a specified sense) and some additional properties (tracking, disturbance rejection, optimality, strict properness,...) are fulfilled.

We suppose a two-term controller feedback system with two exogenous inputs *v* and *w* depicted in Fig.1 and described by the relations

$$y = \frac{B}{A} u + v$$
$$u = \frac{R}{P} w - \frac{Q}{P} y \tag{4}$$

Here, $e = w - y$ is the tracking error, w is the reference and v is a disturbance. It is clear that by putting R=Q we get the traditional one-term feedback system actuating on the output error.

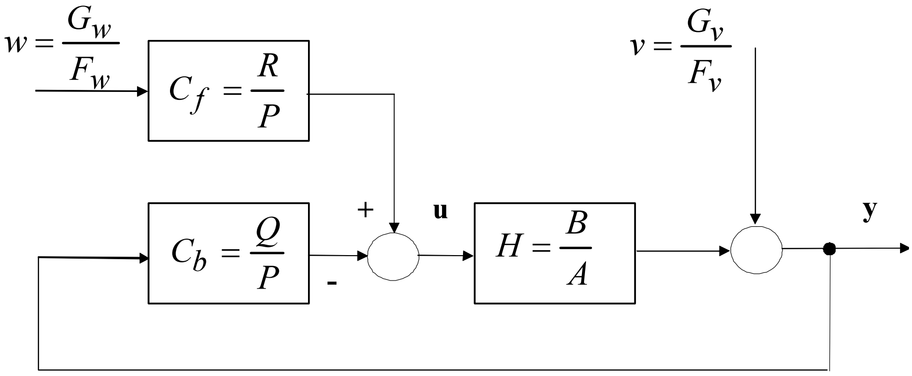

Fig. 1: Two-term control system for reference tracking and disturbance rejection

Roughly speaking, a system is BIBO (bounded input-output) stable if any bounded input produces a bounded output (see e.g. Kucera, 1984; Morari and Zafiriou, 1989; Vidyasagar, 1985). One important result is that system (1) is BIBO stable if and only if H belongs to $\mathbf{R}_{PS}$. A second result is that feedback system (4) is BIBO stable if and only if the common denominator of all transfer functions A P + B Q is a unit of $\mathbf{R}_{PS}$. The third fundamental result says that all stabilizing controllers can be expressed as parameter solutions of

$$\begin{aligned} AP+BQ &= 1 \\ F_w S+BR &= 1 \end{aligned} \tag{5}$$

Diophantine equation (5) is often called Bezout identity and all feedback controllers Q/P are given as

$$\frac{Q}{P} = \frac{Q_0 - AT}{P_0 + BT} \tag{6}$$

where $P_0, Q_0 \in \mathbf{R}_{PS}$ are a particular solution of (5) and T is an arbitrary element of $\mathbf{R}_{PS}$ such the denominator of (6) is non zero.

A mathematical (and even linear) model seldom coincides perfectly with the behaviour of a physical system. So the design of stabilizing controllers for imprecisely known plants is a topic of principal importance.

Now, consider the neighbourhood family of perturbed plants (3). The relevant question can be stated as follows: What part of all stabilizing controllers (5), (6) also stabilize (3) ?

The answer can be found in e.g. Doyle, Francis and Tannenbaum, (1992), Kučera, (1993), Vidyasagar, (1985) and it says: Controllers (5),(6) will BIBO stabilize perturbed plants (3) if

$$\varepsilon_1\|P\| + \varepsilon_2\|Q\| \leq 1 \qquad (7)$$

Remark 1: In the case of stable plants (1) A(s)=1 and B(s)=b(s)/a(s) can be chosen and consequently eq.(6) has a trivial solution $P_0 = 1; Q_0 = 0$. All stabilizing controllers are then given by

$$\frac{Q}{P} = \frac{T}{1-HT}; \quad T \in \mathbf{R}_{PS} \qquad (8)$$

which corresponds to results in Doyle, Francis and Tannenbaum, (1992).

However, the aim is not only restricted to achieve robust stability but also asymptotic tracking, disturbance rejection and potentially other specifications on controller structures (e.g. strict properness). For this reason, parameterized solutions of (6) and the feedforward term R/P in the scheme of Fig.1 are selected. In the framework of the algebraic design, the performance requirements are expressed through divisibility conditions in the underlying ring, and these latter are solved via diophantine equations or appropriate choices of their solutions.

4 ASYMPTOTIC TRACKING AND DISTURBANCE REJECTION

Consider the feedback system in Fig.1, the plant and controller governed by (1), (4) with all stabilizing controllers (5), (6). Moreover, the objective is to design a family of terms R/P such that the plant output y asymptotically tracks the reference signal w expressed in $\mathbf{R}_{PS}$ by the fraction

$$w = \frac{G_w}{F_w} \qquad (9)$$

with G_w unspecified. The tracking error is

$$w - y = \left(1 - \frac{BR}{AP+BQ}\right)\frac{G_w}{F_w} \qquad (10)$$

Since AP+BQ=1 for asymptotic tracking F_w must divide (1-BR) in $\mathbf{R}_{PS}$. Then lemma 1 is deduced.

Lemma 1: Given plant (1), controller system (5) and reference (9). Then the controlled system (in Fig.1) is internally BIBO stable and the plant output asymptotically tracks the reference (9) if and only if A,B and B, F_w are relatively prime in $\mathbf{R}_{PS}$ and transfer functions P,Q,R are given by all solutions of the following equations:

$$\begin{aligned} A\,P \quad + \quad BQ \quad &= \quad 1 \\ F_w S \quad + \quad B\,R \quad &= \quad 1 \end{aligned} \tag{11}$$

Proof: see Kučera, 1984.

Naturally, solutions of the second equation in (11) can be also parameterized in a similar way as in (6). For the controller synthesis, only the expression for R /P is important:

$$\frac{R}{P}=\frac{R_0 - F_w Z}{P_0 + BT} \quad \text{where } Z \text{ is free in } \mathbf{R}_{PS}\,. \tag{12}$$

Now, robust reference tracking is required. It means that the propositions of lemma 1 hold even if the plant H is perturbed in the sense of (3), (4) and $A'P+B'Q$ is still a unit in $\mathbf{R}_{PS}$. The actual reference error can be easily deduced as:

$$w - y = \frac{1}{A'P + B'Q}\left[A'P + B'(Q-R)\right]\frac{G_w}{F_w} \tag{13}$$

Hence for robust reference tracking F_w must divide P as well as (Q-R) because A′, B′, G_w are unspecified precisely. The design can be slightly simplified by the following lemma.

Lemma 2: Let a couple of diophantine equations (11) have a solution. Then, if F_w divides P then F_w divides also (Q-R).

Proof: Let F_w divide P then $P = P_0 F_w$ and subtract the second equation in (11) from the first one. It gives

$$B(Q-R) = F_w(S - AP_0) \tag{14}$$

Since F_w and B are coprime, then from (14) it follows that F_w divides *(Q-R)*.

For the disturbance rejection, we suppose the corrupting disturbance of the form $v = \frac{G_v}{F_v}$ with G_v unspecified and we have to find a part of feedback controllers Q/P such that the effect of disturbance v is asymptotically eliminated from the output in the robust sense. It is evident that

$$y = \frac{A'P}{A'\,P + B'\,Q}\;\frac{G_v}{F_v} \tag{15}$$

The only possibility is to ensure the divisibility of P by F_v. This fact is expressed by the following lemma.

Lemma 3: Let the plant (1), feedback system (4) with *R=0* and a disturbance $v = \dfrac{G_v}{F_v}$ be given, the controller *Q/P* ensures the robust disturbance rejection if and only if *A´P+B´Q* is a unit in $\mathbf{R}_{PS}$ and (5) holds with *P* divisible by F_v .

Proof: follows directly from (15).

A control designer must often fulfil several aims of control systems simultaneously. Now, our aim is to stabilize the feedback system (4) for a family of perturbed plants (3) such that the designed controllers Q/P and R/P simultaneously ensure asymptotic tracking and disturbance rejection (in the sense of lemma 1 and 2, respectively). For a slightly perturbed system (3),(4) the reference error can be easily derived:

$$w - y = \frac{1}{A'P + B'Q}\left[A'P + B'(Q-R)\right]\frac{G_w}{F_w} + \frac{A'P}{A'P + B'Q}\frac{G_v}{F_v} \qquad (16)$$

Since *A´, B´* are not precisely specified but *A´P+B´Q* is still a unit, it is necessary that *P* be divisible by F_w as well as by F_v. According to lemma 2, also *(Q-R)* is divisible by F_w. This can be formulated by the following theorem.

Theorem: The simultaneous robust asymptotic tracking and disturbance rejection for the family of perturbed systems (3) can be achieved by compensators given by all solutions (11) if and only if *A´P+B´Q* is a unit in [illegible] and P is divisible by the multiple $F_w .F_v$.

This fact falls in with the "internal model principle" which is alluded e.g. in Morari and Zafiriou, 1989.

5 FAMILY OF PID-LIKE CONTROLLERS

To illustrate the proposed methodology and philosophy, we will derive and analyze the PI and PID-like controllers which are induced by first and second order systems, respectively.

5.1 PI-like controllers

As a simplest example, we take a first order system

$$H(s) = \frac{b_0}{s + a_0} = \frac{B(s)}{A(s)}, \quad with B(s) = \frac{b_0}{s + m_0}; \quad A(s) = \frac{s + a_0}{s + m_0}; m_0 \rangle 0 \qquad (17)$$

Our aim is to specify all stabilizing controllers such that

a) the system output will track a stepwise reference in the robust sense

b) in addition, the effect of a harmonic disturbance will be asymptotically eliminated

The denominators for a) and b) are $F_w = \dfrac{s}{s+m_0}$ and $F_v = \dfrac{s^2+\omega^2}{(s+m_0)^2}$, respectively.

a) For the tracking problem, all solutions of (11) can be written as

$$P = 1 + \frac{b_0}{s+m_0}T; \quad Q = \frac{m_0 - a_0}{b_0} - \frac{s+a_0}{s+m_0}T; \quad R = \frac{m_0}{b_0} - \frac{s}{s+m_0}Z \qquad (18)$$

where *T, Z* are arbitrary elements of **R**PS. The simplest but non-robust controller can be obtained for *T=Z=0* and it coincides with the polynomial solution of minimal degree. The control law then yields a PP (proportional in feedback and feedforward parts) controller:

$$u = \frac{m_0}{b_0}w - \frac{m_0 - a_0}{b_0}y \qquad (19)$$

For robust control, F_w divides P for all $T = (-\frac{m_0}{b_0} + \frac{s}{s+m_0}T')$, T' is free in **R**PS and the control law of the minimal (MacMillan) degree is obtained for *T′=Z=0* in the form:

$$u = \frac{m_0}{b_0}w - \frac{2m_0 - a_0}{b_0}y + \frac{m_0^2}{b_0}\int (w-y)d\tau \qquad (20)$$

Taking $S = \frac{m_0}{b_0}$, the feedforward part being strictly proper and the control law is a PI-like form

$$u = -\frac{2m_0 - a_0}{b_0}y + \frac{m_0^2}{b_0}\int (w-y)d\tau \qquad (21)$$

<u>Remark 2:</u> Two- term PI controllers as a particular case of PID ones governed by the control law

$$u = k(\beta w - y) + \frac{k}{T_i}\int (w-y)d\tau \qquad (22)$$

were proposed and investigated e.g. in Åström, Wittenmark 1989; Persson and Åström 1993. In (22) $\beta \in$ <0; 1> is a weighing factor. Note that β can be easily specified by the appropriate choice of the free parameters *T* and *Z* in (18). PI controllers (20), (21) represent choices $\beta = \dfrac{m_0}{2m_0 - a_0}$ and $\beta = 0$, respectively.

b) Moreover, for the robust disturbance rejection, P has to be divisible by F_v . To achieve this condition, take $T' = \frac{\omega^2 - m_0{}^2 - 2m_0 s}{(s+m_0)^2} + \frac{s^2+\omega^2}{(s+m)^2} T'' ; T''$ is again free in $\mathbf{R}_{PS}$. The corresponding minimal degree controller is expressed by the following transfer functions:

$$\frac{Q}{P} = \frac{q_3 s^3 + q_2 s^2 + q_1 s + q_0}{s(s^2+\omega^2)} ; \frac{R}{P} = e.g. = \frac{m_0{}^4}{b_0 s(s^2+\omega^2)} \tag{23}$$

where q_i are to be computed by inserting T' in (22) and $T''=0$ into (18).

5.2 PID-like controllers

Consider a second order system with the transfer function $H(s) = \frac{b}{s^2 + a_1 s + a_0}$ and a stepwise reference w. The aim in this case is to:

a) find all controllers with the feedback part only (R=Q) so that they stabilize the feedback system and achieve asymptotic tracking in the robust sense
b) find all two-term controllers with the same specifications as in a)
c) find the minimal degree controller in b) such that both parts Q/P and R/P are strictly proper

The elements in $\mathbf{R}_{PS}$ for this example are the following

$$A = \frac{s^2 + a_1 s + a_0}{(s+m_0)^2} ; \quad B = \frac{b_0}{(s+m_0)^2} ; \quad F_w = \frac{s}{s+m_0}, \textit{ where } \quad m_0 > 0 \tag{24}$$

Equation (5) yields all solutions in the form

$$\begin{aligned} P &= \frac{p_1 s + p_0}{s+m_0} + \frac{b_0}{(s+m_0)^2} T; \quad \textit{where} \qquad & p_0 &= 3m_0 - a_1; \quad q_0 = \frac{1}{b_0}(m_0{}^3 - a_0 p_0) \\ Q &= \frac{q_1 s + q_0}{s+m_0} - \frac{s^2 + as + a_0}{(s+m_0)^2} T & p_1 &= 1; \quad q_1 = \frac{1}{b_0}(3m_0{}^2 - a_0 - a_1 p) \end{aligned} \tag{25}$$

a) Robust asymptotic tracking with the one-term compensator Q/P is achieved iff F_w divides P and it can be obtained by the choice $T = (-\frac{p_0 m_0}{b_0} + \frac{s}{s+m_0} T')$, T' is free in $\mathbf{R}_{PS}$. Substituting for T and $T'=0$, we get the minimal (output error) controller under the form

$$\frac{Q}{P} = \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s(s+\tilde{p}_0)} ; \tag{26}$$

which can be computed through (24). For completeness, a routine calculation gives

$$\tilde{p}_0 = m_0 + p_0; \qquad \tilde{q}_0 = q_0 m_0 + \frac{p_0 m_0}{b_0} a_0;$$
$$\tilde{q}_1 = q_1 m_0 + q_0 + \frac{p_0 m_0}{b_0} a_1; \quad \tilde{q}_2 = q_1 + \frac{p_0 m_0}{b_0}; \tag{27}$$

Equation (26) is nothing else than the proper (realistic) PID controller well known in the more familiar form

$$C_{PID} = K_P \left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \tau s}\right) \tag{28}$$

b) For robust asymptotic tracking with a two-term compensator, we have to solve both eqs.(11), the first of them has been solved in (25). The second one has all solutions of the form

$$R = \frac{r_0}{s + m_0} - \frac{s}{s + m_0} Z; \qquad where \quad r_0 = \frac{m_0^{\ 2}}{b_0} \quad \text{and Z is again free in } \mathbf{R}_{\mathrm{PS}} \tag{29}$$

To get a feedforward part simple, we can take $Z = \frac{r_0}{b_0}$ and the minimal degree (of both parts Q/P and R/P) control law is then governed by the equation:

$$u'' + \tilde{p}_0 u' = \tilde{r}_0 w - \tilde{q}_2 y'' - \tilde{q}_1 y' - \tilde{q}_0 y; \; where \; \tilde{r}_0 = \frac{m_0^{\ 3}}{b_0} \tag{30}$$

in which a generalization of the PID controller can be recognized.

c) Now, both terms Q/P and R/P should be strictly proper. The parameter solution (25) of the stability equation is used. It is easy to verify that the choice $T = (-\frac{p_0 m_0}{b_0} + \frac{s}{s + m_0} \tilde{q}_2)$ gives Q/P strictly proper under the form

$$\frac{Q}{P} = \frac{\tilde{\tilde{q}}_2 s^2 + \tilde{\tilde{q}}_1 s + \tilde{\tilde{q}}_0}{s(s^2 + \tilde{\tilde{p}}_1 s + p_0)}. \tag{31}$$

By the appropriate substitution, one can obtain the coefficients in the explicit form

$$\tilde{\tilde{p}}_0 = p_0 m + b_0 \tilde{q}_2; \qquad \tilde{\tilde{q}}_0 = \tilde{q}_0 m_0;$$
$$\tilde{\tilde{p}}_1 = m_0 + \tilde{p}_0; \qquad \tilde{\tilde{q}}_1 = \tilde{q}_1 m_0 + \tilde{q}_0 - a_0 \tilde{q}_2; \tag{32}$$
$$\tilde{\tilde{q}}_2 = \tilde{q}_1 + \tilde{q}_2 m_0 - a_1 \tilde{q}_2;$$

The term R/P is strictly proper from b). However, the simplest term R is obtained for $Z = \frac{m_0^2 r_0}{s(s^2 + \tilde{\tilde{p}}_1 s + \tilde{\tilde{p}}_0)}$ which gives the following ratio R/P:

$$\frac{R}{P} = \frac{m_0^2 r_0}{s(s^2 + \tilde{\tilde{p}}_1 s + \tilde{\tilde{p}}_0)}; \tag{33}$$

Finally, the strictly proper two terms controller (of minimal degree) is driven by the following differential equation of third order:

$$u''' + \tilde{\tilde{p}}_1 u'' + \tilde{\tilde{p}}_0 u' = \tilde{\tilde{r}}_0 w - \tilde{\tilde{q}}_2 y'' - \tilde{\tilde{q}}_1 y' - \tilde{\tilde{q}}_0 y \tag{34}$$

Remark 3: Note that the proposed controllers are not optimal in the sense of any criteria. However, by parameter $m_0 > 0$ it is possible to influence the robustness of proposed regulators. Indeed, the „most robust“ controller in the sense of the sensitivity function can be easily found through scalar optimization of this parameter.

6 COMPARISON WITH ANOTHER CONTROL PRINCIPLES

The basic CT design can be, of course, performed also in the ring of polynomials. If one wants the control system to have its finite poles located at given positions (pole-placement problem) so the synthesis (with two term compensator) can be expressed by a couple of polynomial diophantine equations:

$$\begin{aligned} a(s)p(s)+b(s)q(s)&=m(s) \\ f(s)t(s)+b(s)r(s)&=m(s) \end{aligned} \tag{35}$$

with m stable polynomial of sufficiently high degree. Eqs. (35) have a solution if the pairs *a, b* and *f, b* are coprime and all solutions are given in a similar way as in other rings. For Example 1 with transfer function (17) one can write

$$p = 1 + b_0\, t; \quad q = \frac{m_0 - a_0}{b_0} - (s + a_0)\, t; \quad r = \frac{m_0}{b_0} - \quad s\, z \tag{36}$$

where *t, z* are free in **P**. However, only the particular solution for $t = z = 0$ can be used since all others are not proper. This solution coincides with (19) and it is not robust, however. Thus, the control synthesis for asymptotic tracking and disturbance rejection in **P** must be solved by a different formulation. The conditions of control aims have to be formulated "a priori" and then the design equations can be set up. Moreover, a control designer must be aware of the correct choice of the degree of the polynomial at the right side of equations. (35).

In the previous sections, we outlined that the divisibility conditions following from robust requirements can be interpreted as the IMC principle suggested e.g. in

Morari and Zafiriou (1989). Now, we precise this relationship. Two-term (or two-degree of freedom) controllers are efficiently applied when good tracking and disturbance rejection are required and dynamic characteristics of these two inputs are substantially different. The IMC scheme of two terms controllers is depicted in Fig.2.a. It is easy to find that the scheme in Fig.2.b leaves the signals u and *y* unaffected. In Figs.2 H_p is an internal model of the controlled system and *M* is an IMC compensator. Since the feedforward term C_f does not influence stability we suppose for a moment *w=0*. Then the relation between the feedback term C_b and the IMC compensator M for the internal stability analysis can be obtained in the form

$$C_b = \frac{M}{1 - H_p M} \tag{37}$$

or conversely

$$M = \frac{C_b}{1 - H_p C_b} \tag{38}$$

Through (37), (38), we have a tool for investigating stability in the sense of IMC (Morari and Zafiriou , 1989). Relation (37) expresses the parametrization of all controllers for an arbitrary but stable transfer function M. The standard choice for M is a tandem of a low-pass filter and a "stable inverse" of the controlled plant transfer function.

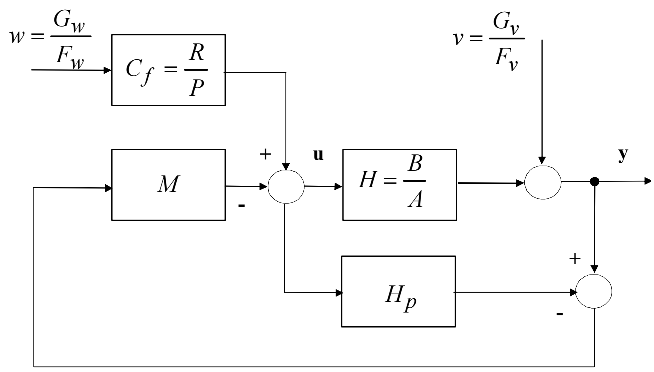

Fig. 2.a: Basic scheme for Internal Model Control systems

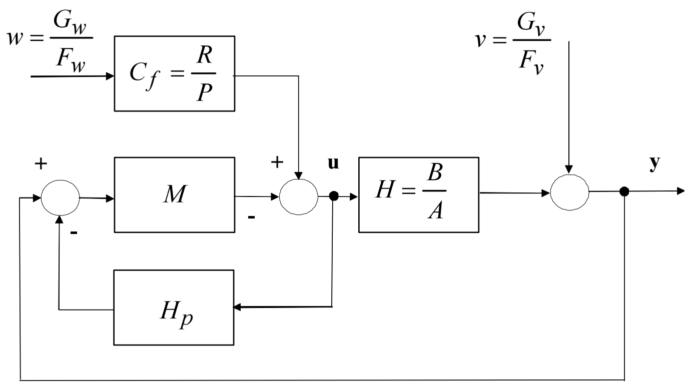

Fig. 2.b: Alternative representation of the IMC structure - relation between IMC and feedback controllers

For a deeper insight into robustness we recall the notions of sensitivity and complementary sensitivity functions (e.g. Doyle, Francis and Tannenbaum, 1992). They are respectively defined

$$\in = \frac{y}{v} = \frac{1}{1 + HC_b} = A(P + BT) \tag{39}$$

$$\eta = 1 - \in = \frac{HC_b}{1 + HC_b} = B(Q - AT) \tag{40}$$

Relations (39), (40) come from the natural idea that $\in$ is the sensitivity of the closed loop transfer function $T_{y,w} = \frac{y}{w}$ to an infinitesimal perturbation in *H*. However, the complementary sensitivity function is no more equal to the mentioned transfer function but is:

$$T_{y,w} = \frac{y}{w} = \in HC_f \tag{41}$$

Naturally, the equation $\in + \eta = 0$ holds and is exact the equation of internal stability (5). It is well known that the relation $\in \approx 0 \eta \approx 1$ cannot be achieved in the whole frequency range(see Doyle, Francis and Tannenbaum, 1992) and the relation

$$\eta \approx 1 \quad \textit{at low frequency } \omega \langle \omega_b \tag{42}$$

is required for good tracking.

7 STABILITY ANALYSIS AND CONTROLLER TUNING

In this section we show that the parameter $m_0 > 0$ defined in the stable factorization (1) can be a proper and effective "tuning knob" for achieving robust stability as well as the control performance. The objective of practical control design is not merely to stabilize a given plant (or a family of perturbed plants) but to place the closed-loop poles in some pre-specified region. For this purpose the left-half plane can be replaced by a more specific domain of stability, namely D={ s: Re s < - m_0 , m_0 > 0}. It can be easily showed (Vidyasagar, 1985) that the set $\mathcal{S}$ (proper and Hurwitz-stable rational functions in D) is a commutative ring and m_0 represents a means to modify the stability region and margin. Further, the value of m_0 influences (unfortunately in a nonlinear way) the boundary for the perturbed plants for which the proposed controller ensures stability. It follows directly from the parameterizations (6), (12) and from the conditions of robust stability (7).

The way in which the control designer can specify and use the right choice of m_0 is explained in tuning the simplest PI (generalized) controller (20) with the feedback part:

$$\frac{Q}{P} = \frac{q_1 s + q_0}{s}; q_1 = \frac{2m_0 - a_0}{b_0}; q_0 = \frac{m_0^{\,2}}{b_0} \tag{43}$$

Naturally, the PI structure is induced by the first order system (17) while the PID structure is based on the second order one. According to Persson and Aström, (1993), we suppose the following models frequently encountered in process industry:

$$H_1 = \frac{e^{-\Theta s}}{2s+1}; \qquad H_2 = \frac{e^{-\Theta s}}{s-1}$$
$$H_3 = \frac{1}{(s+1)^8}; \qquad H_4 = \frac{1}{(s+1)^3(10s+1)} \tag{44}$$

Models (44) are perturbed systems with nominal models H_i^N . For the first two plants the nominal models are simply given by:

$$H_1^N = \frac{1}{2s+1}; \qquad H_2^N = \frac{1}{s-1} \tag{45}$$

Since the plants H_3 , H_4 are of higher order, we have to find beforehand the appropriate nominal plants, then to design controllers, and finally to investigate the behaviour of proposed controllers with perturbed plants. The task of finding simplified (lower order models) is known as a model reduction. Successful control requires a good approximative model of the control plant. According to Isaksson and Graebe, (1993), we utilize probably the simplest way of the model reduction for H_3, H_4 . This approach simply neglects higher order coefficients in the denominator polynomials H_3 , H_4 . We get the following reduced models considered as the nominal plants:

$$H_3^N = \frac{1}{8s+1} = \frac{0.125}{s+0.125}$$
$$H_4^N = \frac{1}{13s+1} = \frac{1/13}{s+1/13} \tag{46}$$

Now, the controller tuning can be formulated as follows: Find the relations between m_0 and the uncertainty region so that the control system is internally stable and determine the maximum value of m_0 for stability. Recall that the infinity norm of the first order transfer functions is given (e.g. Doyle, Francis and Tannenbaum, 1992):

$$\left\| \frac{q_1 s + q_0}{p_1 s + p_0} \right\|_\infty = \max\left\{ \left|\frac{q_1}{p_1}\right|; \left|\frac{q_0}{p_0}\right| \right\} \tag{47}$$

For the couples $H_1, H_1^N \, and H_2, H_2^N$ we solve the task of choosing m_0 for the unknown time delay Θ only. In both cases the uncertainty is given by:

$$\left\| B_i - B_i^{\ N} \right\| = \left\| \frac{\Theta s}{s + m_0} \right\| = \Theta > 0 \quad \text{i=1;2} \tag{48}$$

where the approximation $e^{-\Theta s} \approx (1 - \Theta s)$ was used. The first order Padé approximation $\dfrac{1 - \frac{\Theta s}{2}}{1 + \frac{\Theta s}{2}}$ could be also used but it always increases the final order of the approximated transfer function. We show that the Taylor approximation can give appropriate conditions in analysis and good simulation results. From (7) and (48) immediately follows

$$\Theta \ \max\left\{ \left| \frac{q_1}{p_1} \right| ; \left| \frac{q_0}{p_0} \right| \right\} = \Theta \ \max\left\{ \left| \frac{2m_0 - a_0}{b_0} \right| ; \left| \frac{m_0}{b_0} \right| \right\} \leq 1 \tag{49}$$

after the substitution for a_0, b_0 then relation (49) yields:

$$\begin{aligned} &0 < m_0 \leq \frac{1}{\Theta} && \textit{for} \quad H_1 \\ &0 < m_0 \leq \frac{1}{2}\left(\frac{1}{\Theta} - 1\right) && \textit{for} \quad H_2 \end{aligned} \tag{50}$$

From the second inequality of (50) it clearly follows that the unstable plant $H_2 \, with \Theta > 1$ cannot be stabilized by a PI controller which confirms the fact mentioned in Persson and Åström (1993). Time responses for the closed loop system with $H_1 (with\, \Theta = 1)\ for\ m_0 = 0.4; 0.6$ are shown in Fig.3.a and 3.b, respectively.

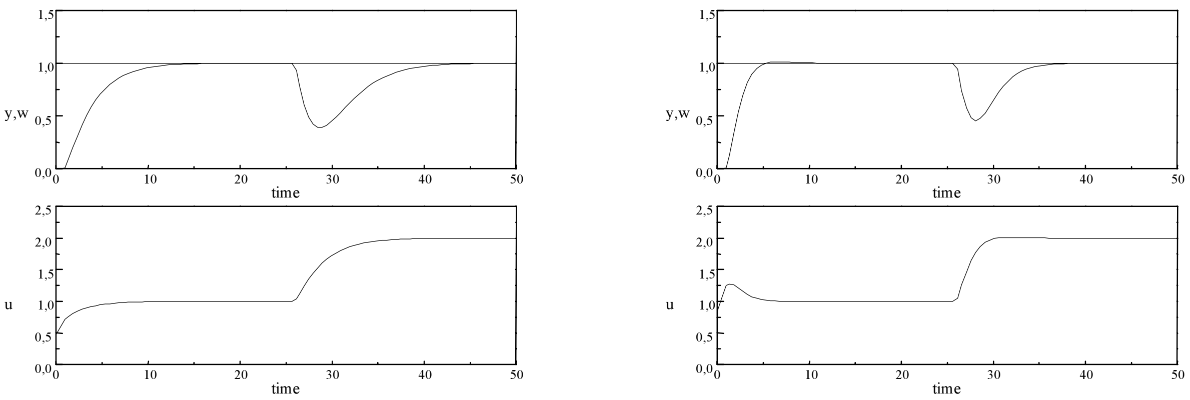

Fig. 3: (*a*) Time response of H_1 by a PI controller for m₀ = 0.4; (*b*) Time response of H_1 by a PI controller for m₀ = 0.6

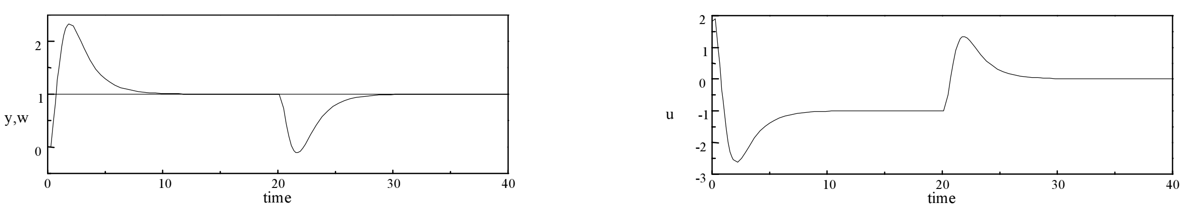

Fig. 4: Time response of H_2 by a PI controller for m_0 = 0.8

The control behaviour of the unstable system H_2 with $a_0 = 1, b_0 = 1, \Theta = 0.3$ is illustrated in Fig.4 for *m_0= 0.8*. If we admit the uncertainty in all three parameters a_0, b_0, Θ the condition of robust stability gives the nonlinear condition for *m_0*:

$$\frac{1+\Theta a_0 \frac{b}{b_0} - \sqrt{(1+\Theta \frac{b}{b_0})^2 - 8|a_0 - a|\Theta \frac{b}{b_0}}}{2\Theta \frac{b}{b_0}} < m_0 < \frac{1+\Theta a_0 \frac{b}{b_0} + \sqrt{(1+\Theta \frac{b}{b_0})^2 - 8|a_0 - a|\Theta \frac{b}{b_0}}}{2\Theta \frac{b}{b_0}} \quad (51)$$

For $a_0 = a > 0, \quad b_0 = b > 0,$ (51) is clearly simplified as the first inequality of (50).

For the model reduction of H_3, H_4 it is not difficult to estimate the norms:

$$h_3 = \left\| H_3^{\ N} - H_3 \right\| = \left\| \frac{-s^8 - 8s^7 - 28s^6 - 56s^5 - 70s^4 - 56s^3 - 28s^2}{(s+1)^8 (8s+1)} \right\| \approx 0.68$$
$$h_4 = \left\| H_4^{\ N} - H_4 \right\| = \left\| \frac{-10s^4 - 31s^3 - 33s^2}{(s+1)^3 (10s+1)(13s+1)} \right\| \approx 0.21 \quad (52)$$

By substituting into stability conditions (7), we get

$$h_i \|Q\| = h_i \max\left\{ \frac{|2m_0 - a_0|}{b_0}; \frac{m_0}{b_0} \right\} \le 1; \quad i = 3,4 \quad (53)$$

For H_3 $(b_0 = a_0 = 0.125)$ *and* H_4 $(b_0 = a_0 = 1/13)$, *it gives*

$$0 < m_0 < 0.15 \quad (54)$$
$$0 < m_0 < 0.18, \quad \textit{respectively.}$$

The time responses of control behaviour are shown in Fig.5 (m_0=0.1) and Fig.6 (m_0=0.12).

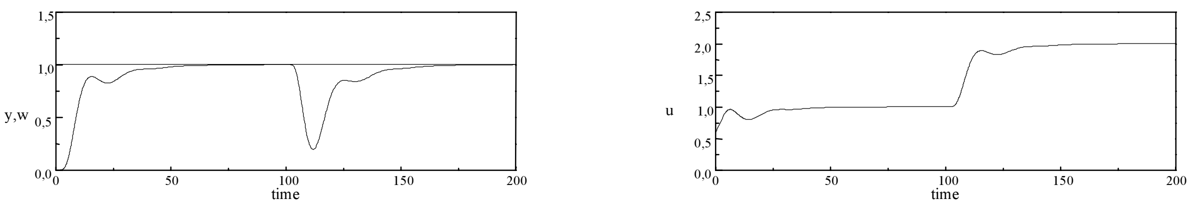

Fig. 5: Time response of H_3 by a PI controller for m_0 = 0.1

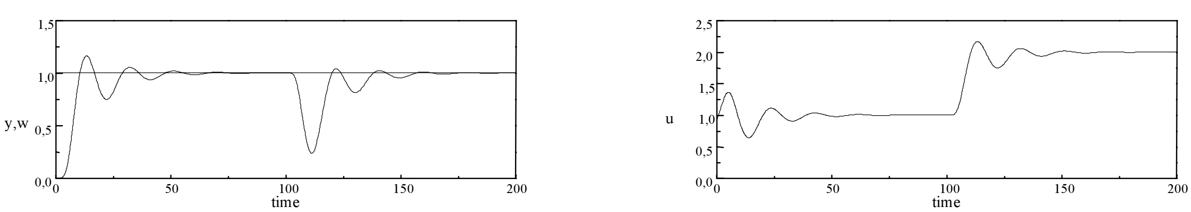

Fig. 6: Time response of H_4 by a PI controller for m_0 = 0.12

A bit more complex situation occurs with PID controllers. For transfer functions H_3, H_4 nominal plants are now as follows

$$H_3^N = \frac{1}{28s^2 + 8s + 1}$$
$$H_4^N = \frac{1}{33s^2 + 13s + 1} \qquad (55)$$

The control responses of original systems are shown in Fig.7 (m_0=0.2) and Fig.8 (m_0=0.35).

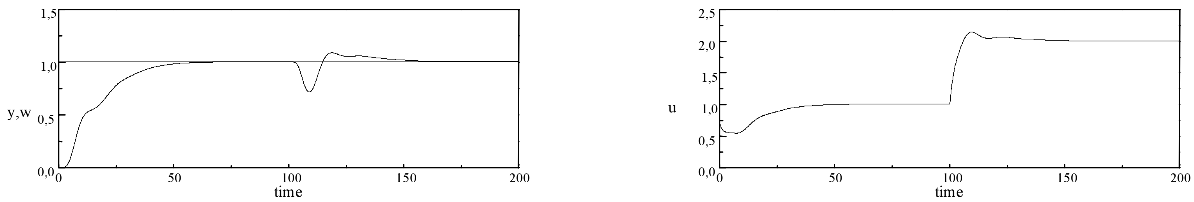

Fig. 7: Time response of H_3 by a PID controller for m_0 = 0.17

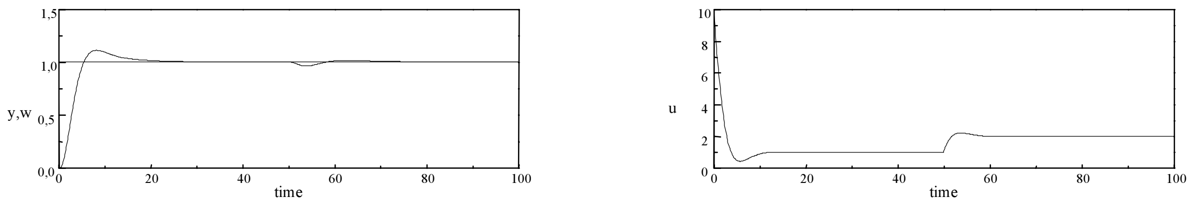

Fig. 8: Time response of H_4 by a PID controller for m_0 = 0.35

Fig.9 highlights the significance of m_0 as a knob of the "modulus margin". Nyquist plots of the open loop transfer function H_3 and a PI controller for $m_0$0.12; 0.15 and 0.2 indicate the distance from the critical point. These curves confirm the stability region given by (54) and the fact that decreasing m_0 causes increasing robustness.

It is highly desirable to have a tuning parameter which the user can change to influence the properties of the closed loop system in a predictable way. The free parameter m_0 in the coprime factorization (1) specifies always the closed loop poles, or more precisely, the zeros of the controller (see e.g. (43). Decreasing m_0 gives a slower but more robust control with less overshoot.

In a control practise, a control designer has frequently to set the control parameters in some non-optimal but acceptable way. From the discussion, it follows that, in the case of totally unknown parameters, it is necessary to start with a very small parameter m_0. If the control response is stable, the value of m_0 can be carefully increased. The automatic tuning of m_0 is still an unsolved problem.

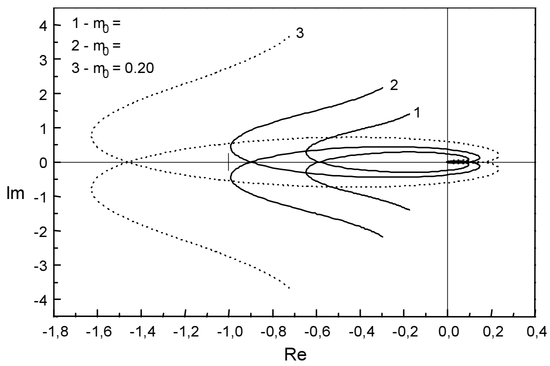

Fig. 9: Nyquist plots of the open loop transfer function H_3 and a PI controller for m_0 = 0.12; 0.15 and 0.2

8 INTERVAL SYSTEMS AND POLYTOPES

Systems with parametric uncertainties can be often described by three basic types of uncertain models with the following hierarchy: $\text{interval} \subset \text{affine} \subset \text{multilinear}$. For more details see Barmish (1994). In this part interval and affine uncertainty structures will be considered and analysed.

Interval systems are a natural class of uncertain systems. They are described as a ratio of two polynomials where parameters are assumed to lie within specified intervals. Uncertainties in every coefficient must be independent. The interval system transfer function is then addressed as:

$$G(s,b_j,a_i) = \frac{B(s,b_j)}{A(s,a_i)} = \frac{\sum_{j=0}^{m}\left[b_j^-;b_j^+\right]s^j}{\sum_{i=0}^{n}\left[a_i^-;a_i^+\right]s^i}; \quad m < n \tag{56}$$

where b_j^-, b_j^+, a_i^- and a_i^+ are specified lower and upper bounds of the j-th perturbation b_j, and i-th perturbation a_i, respectively.

More general class of systems with parametric uncertainty is known as affine systems. An uncertain polynomial

$$p(s,q) = \sum_{i=0}^{n} a_i(q)s^i \tag{57}$$

is said to have an affine linear uncertainty structure if each coefficient function $a_i(q)$ is an affine linear function of *q*. In other words, affine uncertain polynomials linearly depend on uncertain parameters and each parameter may occur simultaneously in several coefficients. Uncertain polynomials (57) constitute polytopes from the set of polynomials. They can be always expressed in the form:

$$p(s,q) = p_0(s) + \sum_{i=1}^{n} q_i p_i(s) \quad (58)$$

where $q_i \in [q_i^-; q_i^+]$. Hence, a polytope of polynomials is depicted by the $(n+1)$ polynomials $p_0(s)$, $p_1(s)$, ..., $p_n(s)$ with n parameter bounding intervals. More generally, an uncertain rational function is said to have an affine linear uncertainty structure if both polynomials, in numerator and denominator, have affine linear uncertainty structures.

8.1 Robust stability analysis

A characteristic polynomial of a closed loop connecting a plant with linear affine uncertain structure and fixed controller has the form (58) and according to proposed robust control design can be obtained from

$$c(s, b_j, a_i) = num(AP + BQ) \quad (59)$$

Relation (59) means the numerator of the rational function *AP+BQ*. If this polynomial is stable not only for nominal values of uncertain parameters or particular set of them but for all values $q_i \in [q_i^-; q_i^+]$ (or in this case $b_j \in [b_j^-; b_j^+]$ and $a_i \in [a_i^-; a_i^+]$), then it is called robustly stable. It is necessary to remark that there can the problems with so-called degree dropping, so it is very often supposed characteristic polynomial with invariant degree for any $q_i \in [q_i^-; q_i^+]$.

The very important and effective conception used for testing of robust stability is the value set concept. Given an uncertain polynomial $p(s,q)$ and an uncertainty bounding set Q, then, at a fixed frequency $\omega \in R$, the value set is the subset of the complex plane consisting of all values which can be assumed by $p(j\omega, q)$ as q ranges over Q. Said another way, $p(j\omega, Q)$ is the range of $p(j\omega, \cdot)$.

The value set concept is applied in the following fundamental theorem which is known as the zero exclusion condition. Suppose that a family of polynomials $P = \{p(\cdot, q) : q \in Q\}$ has an invariant degree with associated uncertainty bounding set Q which is path wise connected continuous coefficient functions $q_i(q)$ for i = 0,1,2,...,n and at least one stable member $p(s, q^0)$. Then P is robustly stable if and only if the origin, z = 0, is excluded from the value set $p(j\omega, Q)$ at all frequencies $\omega \geq 0$; i.e., P is robustly stable if and only if $0 \notin p(j\omega, Q)$ for all frequencies $\omega \geq 0$. Consequently, the practical test is quite simple. Roughly speaking, if the family contains a stable member and if the value set excludes the point 0 for all frequencies, then the family is concluded to be robustly stable.

Formal definition of the value set concept, more general version (applicable also for discrete-time systems or other stability regions) of the zero exclusion condition and subsequent results can be find e.g. in Barmish (1994).
Robust stability analysis for interval polynomials (with independent coefficients) is simpler and it can be performed with through the Kharitonov's theorem. This fundamental tool states that the stability of an interval polynomial can be determined by testing the stability of four polynomials which can be easily obtained using upper and lower values of the uncertain parameters. Let us consider a real interval polynomial of invariant degree *n* as:

$$I(s,i_j)=\sum_{j=0}^{n}\left[i_j^-;i_j^+\right]s^j \tag{60}$$

The interval polynomial (60) is Hurwitz stable if and only if the following four polynomials (Kharitonov's polynomials) are Hurwitz stable:

$$\begin{aligned}
K_1(s)&=i_0^+ + i_1^+ s + i_2^- s^2 + i_3^- s^3 + \cdots\\
K_2(s)&=i_0^- + i_1^- s + i_2^+ s^2 + i_3^+ s^3 + \cdots\\
K_3(s)&=i_0^+ + i_1^- s + i_2^- s^2 + i_3^+ s^3 + \cdots\\
K_4(s)&=i_0^- + i_1^+ s + i_2^+ s^2 + i_3^- s^3 + \cdots
\end{aligned} \tag{61}$$

There are two basic possibilities for robust stability analysis in the case of uncertain polynomial with affine linear structure. First option is to "overbound" original more general and complicated structure by the simple interval polynomial. Thus, the parameters dependence is ignored. Then, by testing the stability of the "overbounding" interval polynomial we obtain sufficient condition for the stability of the original uncertainty structure. The second alternative is to use some more general tools. Typical robust stability analysis instruments are e.g. the edge theorem, the thirty-two edge theorem and in special cases (for first order compensator) the sixteen plant theorem. For more details and references to related literature see Barmish (1994) or another robust control textbook.

Practical robust stability checking and depiction of graphical interpretation can be very easily and comfortable done with assistance of Polynomial Toolbox for Matlab – some basic routines description can be found in Šebek, Hromčík, Ježek (2000) or see directly web page www.polyx.com.

8.2 Illustrative example

A controlled system is assumed to be given by a second order interval transfer function:

$$G(s,b_j,a_i)=\frac{b_0}{s^2+a_1 s+a_0} \tag{62}$$

where $b_0 = a_1 = a_0 \in [0.5; 1.5]$.

The system (62) with parameters $b_0 = a_1 = a_0 = 1$ is supposed as a nominal one. Both 1DOF or 2DOF asymptotic tracking controllers of a PID-like type were designed and tuned according to algebraic method given in part 5.2. The feedback controller is supposed to have the form:

$$C_b(s) = \frac{Q}{P} = \frac{q_2 s^2 + q_1 s + q_0}{p_2 s^2 + p_1 s} \tag{63}$$

It is obvious that the closed-loop characteristic polynomial has the affine linear structure:

$$c(s, b_j, a_i) = d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 \tag{64}$$

where $d_4 = p_2$ $\quad d_3 = a_1 p_2 + p_1$

$d_2 = a_0 p_2 + a_1 p_1 + b_0 q_2 \quad d_1 = a_0 p_1 + b_0 q_1 \quad d_0 = b_0 q_0$

The following procedure was used for all closed-loop and step responses simulations: each uncertain parameter in interval plant (62) was divided into 5 partial intervals (6 values) and all possible combinations were computed. Thus, the total number $6^3 = 216$ certain systems from the infinite systems set (62) was obtained. Fig. 10 shows step responses of this "controlled" systems set (CSS).

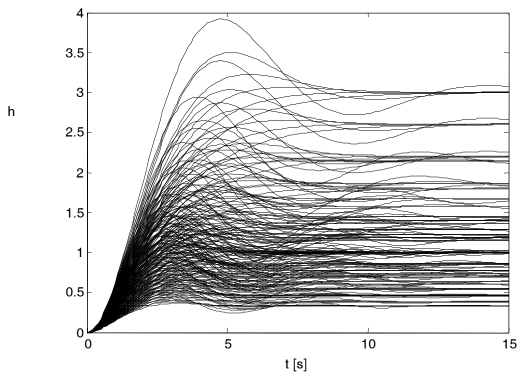

Fig. 10: Step responses of CSS

The first choice of tuning parameter $m = 0.5$ gives the feedback controller coefficients for nominal system in (63) $q_2 = q_1 = -0.5$; $q_0 = 0.0625$; $p_2 = p_1 = 1$. Now, the question is whether the designed controller keeps the closed loop stable for all possible combinations of uncertain parameters in (62). The robust stability can be tested using the Kharitonov's theorem. The polytope (64) with respective coefficient values can be simply "overbounded" by an interval polynomial. Sequentially, the stability of this polynomial has to be investigated. The value sets (Kharitonov's rectangles) are depicted in Fig. 11 a). To see better what is happening in the neighborhood of the point [0; 0j], the graph in Fig. 11 a) is zoomed – Fig. 11 b). The

Kharitonov's rectangles includes the origin, hence it can not be concluded if (64) itself is robustly stable or not, because we covered the linear affine structure by interval one and ignored the mutual dependence of coefficients in (64).

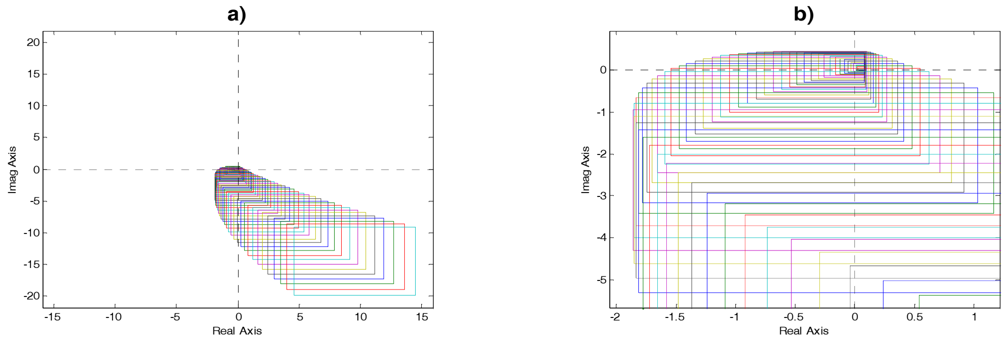

Fig. 11: Kharitonov's rectangles (*m=0.5*)

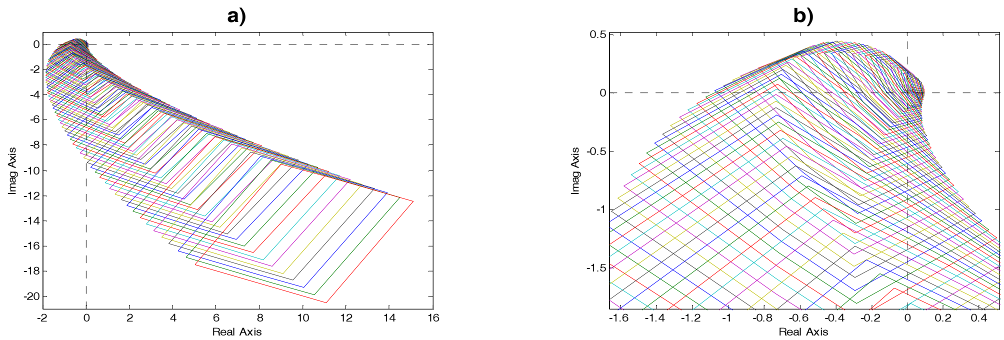

Fig. 12: Value sets (*m=0.5*)

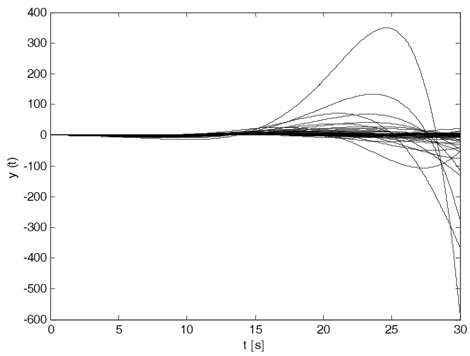

Fig. 13: Closed-loop response of CSS (1DOF; *m=0.5*)

The polytopic value sets can be plotted for robust stability testing of (64) with necessary and sufficient condition – see Fig. 12 a), again with detailed view in Fig. 11 b). For the controller tuned by $m = 0.5$, the closed-loop response is really not robustly stable. This fact is confirmed by Fig. 13 where control behaviour of all 216 transfer functions is shown. It confirms that at least a part of control loops are unstable.
Another value of tuning parameter $m = 1$ generates the 1DOF controller (63) with parameters $q_2 = 2$; $q_1 = q_0 = p_2 = 1$; $p_1 = 3$. The robust stability analysis was done in a similar way as in the previous case. In Fig. 14 and 15 can be seen that

“overbounding” interval polynomial is unstable, however the original polytope of polynomials (64) has stable value sets in the sense of the zero exclusion condition.

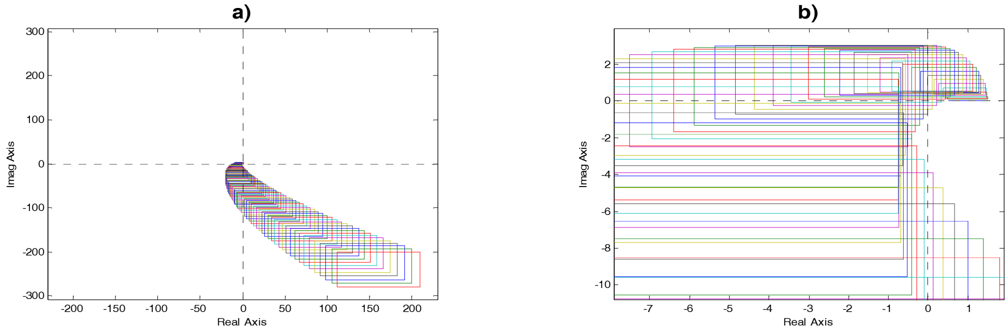

Fig. 14: Kharitonov’s rectangles (*m=1*)

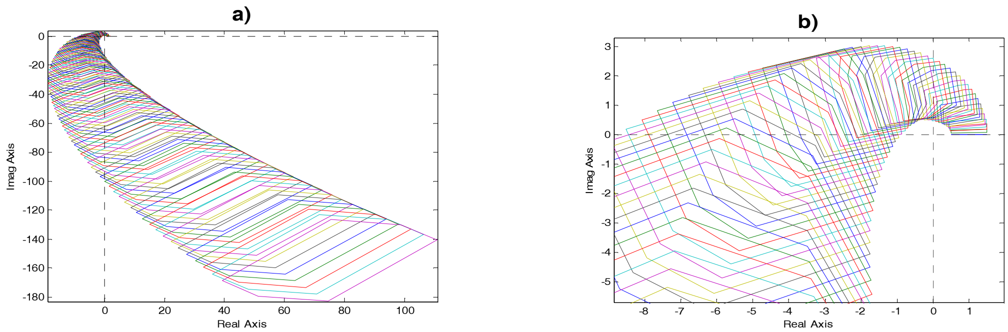

Fig. 15: Value sets (*m=1*)

The closed-loop control behavior for all CSS is stable indeed as shown in Fig. 16.

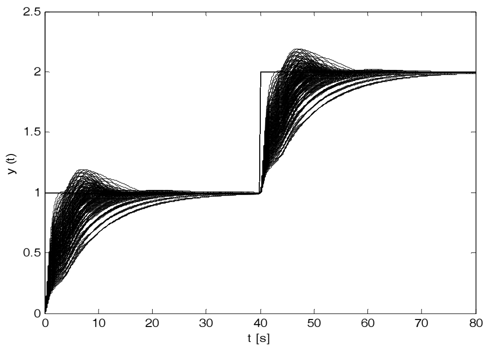

Fig. 16: Closed-loop response of CSS

(1DOF; m=1)

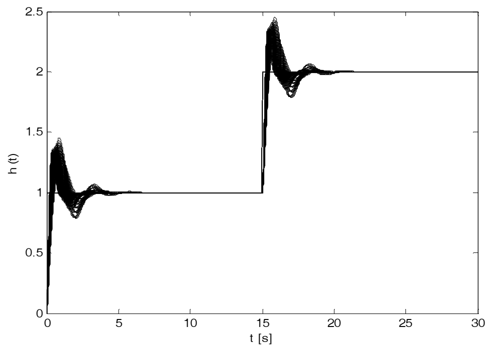

Fig. 17: Closed-loop response of CSS

(1DOF; m=4)

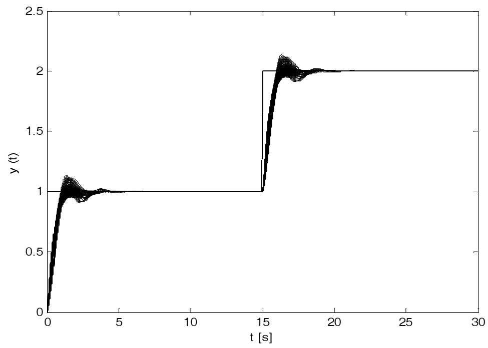

Fig. 18: Closed-loop response of CSS (2DOF; *m=4*)

Further improvement can be obtained by the choice $m = 4$ which gives control responses depicted in Fig. 17 and Fig. 18 for 1DOF and 2DOF control system configurations, respectively.

9 CONCLUSION

The contribution is devoted to the design of simple proper and robust SISO regulators with various and different specifications. The methodology is based on properties of the ring of stable rational functions in the pre specified region of the left half-plane. It is shown that algebraic manipulations (divisibility in the appropriate ring and parameter solutions of diophantine equations) yield a wide diversity of proper regulators. Robust controllers are then chosen from the family of all stabilizing ones. The proposed methodology enables to tune and influence the control behaviour through a single scalar positive parameter. This approach cannot be applied in the ring of polynomials since almost all solutions give controllers without the property of properness. Tuning, model reduction and stability analysis for systems with dead-time are also included for the family of PI and PID- like controllers. Another robust analysis can be studied through the value set concept, zero exlusion theorem and Kharitonov´s theorem. The resulting robust controllers can be successfully applied in adaptive control as well as in the control of uncertain plants. All simulations and visualisations were performed in the Matlab 12 + Simulink and Polynomial toolbox environment.

10 REFERENCES

Åström, K.J., Hägglund, T. (1995): PID Controllers: Theory, Design and Tuning, Instrument Society of America, USA.

Åström, K.J., Wittenmark, B. (1989): Adaptive Control. Addison Wesley, Reading, MA.

Åström, K.J., Hang, C.C., Persson, P. and W.K. Ho (1991): Towards Intelligent PID Control, *Automatica*, Vol. 28, No. 1, pp. 1-9.

Åström, K.J., Hang, C.C., Lim, B.C. (1994): A New Smith Predictor for Controlling a Process with an Integrator and Long Dead-Time. *IEEE Trans. on Autom. Control,* Vol.39, No.2, p.343-345.

Barmish, B.R. (1994): New Tools for Robustness of Linear Systeme, Macmillan, New York.

Doyle, C.D., Francis, B.A., Tannenbaum, A.R. (1992): Feedback Control Theory. Macmillan, NY.

Doyle, C.D., Vidyasagar, M. (1983): Algebraic and Topological Aspects of the Regulator Problem for Lumped Linear Systems. *Automatica,* 19, pp. 87-90.

Isaksson, A.J, Graebe, S.F.: Model Reduction for PID Design. *Prepr. 12th IFAC World Congress*, Sydney, Vol.8, pp.257-262.

Kučera, V. (1979): Discrete Linear Control. Chichester.

Kučera, V. (1984): Design of Internally Proper and Stable Systems. Prepr.9th IFAC World Congress, Budapest, Vol.VIII, pp.94-98.

Kučera, V. (1993): Diophantine Equations in Control - A Survey, *Automatica.* Vol.29, No.6, pp.1361-75.

Morari, M., Zafiriou, E. (1989): Robust Process Control. Prentice Hall, New Jersey.

Persson, P., Åström, K.J. (1993): PID Control Revisited. *Prepr. 12th IFAC World Congress,* Sydney, Vol.8, pp.241-244.

Prokop, R., Dostál, P., Meszáros, A. (1992): Continuous-time Control via proper Regulators. *Journal A*, vol.33, No.4, pp.11-17.

Prokop, R., Mészáros, A. (1996): Design of robust controllers for SISO time delay systems, *Journal of Electrical Engineering, 47*(11-12),287-294.

Prokop, R., Corriou, J.P. (1997): Design and analysis of simple robust controllers, *Int. J. Control, 66*(6), pp. 905-921.

Prokop, R., Dostál, P., Prokopová, Z. (2000): Robust linear controllers for unstable systems with time delay, *Proc. 3rd IFAC Symp. Robust Control Design*, Prague, 171-176.

Rad, A.B., Lo, W.L.: (1994) Predictive PI Controller. Int.J.of Control, vol.60, No.6, pp.953-975.

Ramambason, O.C, Bonvin, D. (1993): Differential Sensitivity and Control System Structures. Prepr. 12th IFAC World Congress, Sydney, Vol.4, pp.39-42.

Salgado, E.M., Goodwin, G.C. and R.H.Middleton (1993): A Constraint Optimization Approach to Robust Control. Proc. of the 32nd Conf. on Dec.and Control, Texas.

Šebek, M., Hromčík M., Ježek, J. (2000): Polynomial toolbox 2.5 and systems with parametric uncertainties, *Proc. 3rd IFAC Symp. Robust Control Design*, Prague.

Venkatashankar, V., Chidambaram, M. (1994): Design of P and PI Controllers for Unstable Firs-order Plus time Delay Systems, Int.J.of Control, Vol.60, No.1, p.137-144.

Vidyasagar, M. (1985): Control System Synthesis: A Factorization Approach. MIT Press, Cambridge, M.A.

Youla, D.C., Bongiorno, J.J. and Jabr, H.A. (1976): Modern Wiener-Hopf Design of Optimal Controllers, Part I. IEEE Trans. on AC, AC-21, pp.14-21.