Brno University of Technology

Faculty of Electrical Engineering and Computer Science

Department of Computer Science and Engineering

RNDr. Alexander Meduna, CSc.

Context-Free Multirewriting

with a Reduced Number of Nonterminals

Extended Abstract of Habilitation Thesis

VUTIUM

Brno 2000

ISBN 80–214–1616–5

Contents

About the Author

Alexander Meduna was born June 27, 1957 in Olomouc, the Czech Republic. From 1978 to 1982, he attended the Palacký University, receiving his M.S. in Computer Science in 1982. He received his Ph.D. in Computer Science at the Brno University of Technology in 1988.

In 1988, based on some of his publications, he was invited to the University of Missouri–Columbia, USA, where he taught several computer science classes for almost a decade. Most of these classes dealt with automata, formal languages, computability, and compilers. In 1996, he returned to the Czech Republic to do research at the Computing Center of the Brno University of Technology. From there, he moved to the Department of Computer Science at the Brno University of Technology in 1999. At this department, he has since taught various computer science classes, such as compilers and theoretical computer science.

1. Introduction

In the formal language theory, the standard context-free rewriting works purely sequentially because ordinary context-free grammars rewrite only one symbol during a derivation step. The present thesis, however, discusses a parallel modification of this rewriting, called *context-free multirewriting*, which simultaneously applies productions to several symbols during a single derivation step.

This parallel modification of context-free rewriting is intensively investigated at present because computational parallelism is central to the modern computer science as a whole. Even more importantly, this modification significantly increases the power of context-free grammars; in fact, most of the grammatical multirewriting models are as powerful as the Turing machines. Thus, it comes as no surprise that the context-free multirewriting is a vivid trend of today's formal language theory.

The thesis concentrates its investigation on the reduction of the number of nonterminals in the context-free multirewriting. Most significantly, it studies how to achieve this reduction without any decreasement of the generative power. By doing so, it actually makes the context-free multirewriting more succinct and economical, and this economization is obviously highly appreciated both from a practical and theoretical viewpoint.

More specifically, the following two types of context-free rewriting are central to this thesis:

(A) scattered rewriting
(B) multisequential and multicontinuous rewriting

(A) The former is performed by grammars based on sequences of context-free productions, by which these grammars simultaneously rewrite several nonterminals during a derivation step. (B) The latter is carried out by grammars that use context-free-like productions that have a terminal or a nonterminal on their left-hand sides. By using extremely simple regular languages, they specify sequences of symbols that are rewritten during a derivation step and, in addition, place some slight restrictions on the context appearing between the rewritten symbols. Otherwise, they work by analogy with context-free grammars.

2. Preliminaries

The present chapter recalls some basic notions and introduces conventions used henceforth. For other notions used in the formal language theory, consult [39].

For an alphabet, W, W^* denotes the *free monoid* generated by W under the operation of concatenation; its unit is called the empty word and denoted by ε. W^+ denotes the *free semigroup* generated by W under the operation of concatenation; that is, $W^+ = W^* - \{\varepsilon\}$. For every $w \in W^*$, $|w|$ denotes the length of w.

A *context-free grammar* is a quadruple

$$G = (V, T, P, S)$$

where

V is G's total alphabet;
T is the terminal alphabet such that $T \subseteq V$;
P is a finite set of productions of the form

$$A \to x$$

with $A \in V - T$ and $x \in V^*$;
S is the start symbol such that $S \in V - T$.

If $A \to x \in P$ and $u, v \in V^*$, then G makes a *derivation step* from uAv to uxv, symbolically written as

$$uAv \Rightarrow uxv$$

Mathematically, $\Rightarrow$ represents a binary relation on V^*; denote its transitive and reflexive closure by $\Rightarrow^*$. If $S \Rightarrow^* w$ and $w \in V^*$, then w is a *sentential form* derived by G. The *language of* G, $L(G)$, is defined as

$$L(G) = \{ w: S \Rightarrow^* w \text{ and } w \in T^*\}$$

Conventions

For every grammar, G, discussed in the thesis, V, T, and S have the above meaning. N denotes G's alphabet of nonterminals, defined as $N = V - T$. Early lowercase letters (a, b, c, …) and late lowercase letters (…, x, y, z) denote members of V and V^*, respectively. Early uppercase letters (A, B, C, …) denote symbols in N.

The families of context-sensitive and recursively enumerable languages are denoted by ***CS*** and ***RE***, respectively.

3. Scattered rewriting

Scattered rewriting is performed by scattered context grammars. These grammars are based on sequences of context-free productions, by which they simultaneously rewrites several nonterminals during a single derivation step (see [4-5], [9], [21], [25], [27-28], [30], [33], [37-39], [41-42], [44], [47] , and [51]).

Definition

Formally, a scattered context grammar is a quadruple

$$G = (V, T, P, S)$$

where V, T, and S have the same meaning as in a context-free grammar, and P is a finite set of productions of the form

$$(A_1, A_2, ..., A_n) \to (x_1, x_2, ..., x_n)$$

where n is a positive integer (by the conventions introduced in Chapter 2, $A_i \in V - T$ and $x_i \in V^*$, for $i = 1, ..., n$). By using $(A_1, A_2, ..., A_n) \to (x_1, x_2, ..., x_n)$, G makes a derivation step so it simultaneously replaces $A_1, A_2, ..., A_n$ with $x_1, x_2, ..., x_n$, respectively; symbolically,

$$u_1A_1u_2A_2\, u_3 \;...\; u_nA_nu_{n+1} \Rightarrow u_1x_1u_2x_2\, u_3 \;...\; u_nx_nu_{n+1}$$

for any u_1 through u_{n+1}. As usual, $\Rightarrow^*$ denotes the transitive and reflexive closure of $\Rightarrow$, and the *language of* G, $L(G)$, is defined as

$$L(G) = \{ w: S \Rightarrow^* w \text{ and } w \in T^* \}$$

The following two special types of scattered context productions play an important role in this chapter.

Consider a scattered context grammar, $G = (V, T, P, S)$, and a production, $(A_1, A_2, ..., A_n) \rightarrow (x_1, x_2, ..., x_n) \in$ P.

- $(A_1, A_2, ..., A_n) \rightarrow (x_1, x_2, ..., x_n)$ is *terminating* if $A_1A_2... A_n \Rightarrow^* x$ in G for some $x \in T^*$;

- $(A_1, A_2, ..., A_n) \rightarrow (x_1, x_2, ..., x_n) \in$ P is *erasing* if $x_i = \varepsilon$ for some $i \in \{1, 2, ..., n\}$.

Example

Consider a scattered context grammar, G, with the following three productions

$$(S) \rightarrow (AA),\ (A, A) \rightarrow (aA, bAa), \text{ and } (A, A) \rightarrow (\varepsilon, \varepsilon)$$

For instance, G derives *aabbaa* as follows

$$S \Rightarrow AA \Rightarrow aAbAa \Rightarrow aaAbbAaa \Rightarrow aabbaa$$

Observe that G generates the following non-context-free language

$$L(G) = \{ a^ib^ia^i: i \geq 0\}$$

Every production in G is terminating. $(A, A) \rightarrow (\varepsilon, \varepsilon)$ is an erasing production while the other two productions are not.

Results

Scattered rewriting is discussed in Chapter 3, which is divided into four sections.

Section 3.1 conceptualizes scattered context grammars.

Section 3.2, consisting of three subsections, discusses the generative power of scattered context grammars. More precisely, by a surprisingly simple proof, Section 3.2.1 demonstrates that these grammars characterize ***RE***. Then, Sections 3.2.2 and 3.2.3 reconsider this characterization in more detail. Specifically, by using a rather complicated proof technique, it establishes this characterization based on the scattered context grammars containing no more than four nonterminals. Section 3.2.3 studies the economy of transformation of phrase structure grammars, which also define ***RE***, to scattered context grammars. More precisely, it describes a transformation that converts any phrase-structure grammar, H, to an equivalent scattered context grammar, G, so that G has no more than five more nonterminals than H.

Section 3.3 discusses the terminating productions, defined in Section 3.1, and their effect to the power of scattered context grammars. It narrows its attention to the scattered context grammars whose sentential forms contain sequences of nonterminals formed by shuffling the left-hand sides of terminating productions. It proves that these grammars cannot even generate some languages in ***CS***, so they are significantly less powerful than ordinary scattered context grammars, discussed in Section 3.2. From this decreasement of the generative power, Section 3.2 derives that the one-nonterminal scattered context grammars are less powerful than the four-nonterminal scattered context grammars.

Section 3.4 deals with the scattered context grammars without erasing productions, defined in Section 3.1. These grammars obviously cannot characterize ***RE*** because the language family that they generate is contained in ***CS*** and ***SC*** $\subset$ ***RE*** (see [9]). However, Section 3.4 introduces some simple language operation and characterizes ***RE*** based on the introduced operations over the family of languages generated by these grammars, which contain no more than five nonterminals.

4. Multisequential and Multicontinuous Rewriting

Multisequential rewriting and multicontinuous rewriting are performed by multisequential grammars and multicontinuous grammars, respectively (see [11]). Both grammars use context-free-like productions that have a terminal or a nonterminal on their left-hand sides. By using extremely simple regular languages, called *selectors*, they specify sequences of nonterminals that are rewritten during a derivation step and, in addition, place some restrictions on the context appearing between the rewritten symbols. Otherwise, they work by analogy with context-free grammars.

4.1 Multisequential Grammars

Definition

A *multisequential grammar*, G, is a quintuple

$$G = (V, T, P, S, K),$$

where V, T, and S have the same meaning as in a context-free grammar. P is a finite set of productions of the form

$$a \rightarrow x$$

(recall that by the conventions introduced in Chapter 2, $a \in V$ and $x \in V^*$). K is a finite set of selectors of the form

$$X_1\textbf{active}(Y_1)\, X_2 \ldots X_n\textbf{active}(Y_n)\, X_{n+1}$$

where

n is a positive integer
for $i = 1, \ldots, n + 1$, $X_i \in \{Z^*: Z \subseteq \mathrm{V}\}$
for $j = 1, \ldots , n$, $Y_j \subseteq V$ and $Y_j \neq \emptyset$

G makes a *derivation step* of the form

$$u_1a_1u_2a_2\ u_3 \ldots u_na_nu_{n+1} \Rightarrow u_1x_1u_2x_2\ u_3 \ldots u_nx_nu_{n+1}$$

if K contains a selector, X_1**active** (Y_1) X_2... X_n**active**(Y_n) X_{n+1}, satisfying

for $i = 1, ..., n + 1$ $u_i \in X_i$
for $j = 1, ... , n$, $a_j \in Y_j$ and $a_j \rightarrow x_j \in P$

The *language of* G, $L(G)$, is defined as

$$L(G) = \{ w: S \Rightarrow^* w \text{ and } w \in T^*\}$$

where $\Rightarrow^*$ denotes the transitive and reflexive closure of $\Rightarrow$.

Example

Consider a scattered context grammar, G, with the follwoing eight productions

$$S \rightarrow AA, S \rightarrow BB, A \rightarrow Aa, A \rightarrow Ba, B \rightarrow Ab, B \rightarrow Bb, A \rightarrow \varepsilon, B \rightarrow \varepsilon$$

and these three selectors

active$(\{S\})$, **active**$(\{A\})\{a, b\}^*$**active**$(\{A\})\{a, b\}^*$, **active**$(\{B\})\{a, b\}^*$**active**$(\{B\})\{a, b\}^*$

For instance, G derives *abab* as follows

$$S \Rightarrow BB \Rightarrow AbAb \Rightarrow AabAab \Rightarrow abab$$

Observe that G generates the following non-context-free language

$$L(G) = \{ ww: w \in \{a, b\}^* \}$$

Results

Multisequential grammars are discussed in Section 4.1, which consists of two subsections. Section 4.1.1 conceptualizes and illustrates these grammars. Then, Section 4.1.2 discusses their generative power; more precisely, it demonstrates that the six-nonterminal multicontinuous grammars characterize ***RE***.

4.2 Multicontinuous Rewriting

Definition

A *multicontinuous grammar*, G, is a quintuple

$$G = (V, T, P, S, K),$$

where V, T, P, and S have the same meaning as in a multisequential grammar. K is a finite set of selectors of the form

$$X_1\mathbf{active}\,(Y_1)\,X_2 \ldots X_n\mathbf{active}(Y_n)\,X_{n+1}$$

where

n is a positive integer
for $i = 1, \ldots, n + 1$, $X_i \in \{Z^*: Z \subseteq V\}$
for $j = 1, \ldots, n$, $Y_j \in \{Z^+: Z \subseteq V \text{ and } Z \neq \emptyset\}$.

For every $v \in V^+$, where $v = a_1 \ldots a_{|v|}$ with $a_i \in V$ for $i = 1, \ldots, |v|$, define the language *ContinuousRewriting*(v) $\subseteq \mathrm{V}^*$ by the following equivalence:

for every $z \in V^*$,

$$z \in \mathit{ContinuousRewriting}(v)$$

if and only if

$$a_i \rightarrow x_i \in P \text{ for } i = 1, \ldots, |v|, \text{ and } z = x_1 \ldots x_{|v|}$$

G makes a derivation step of the form

$$u_1y_1u_2y_2\,u_3 \ldots u_ny_nu_{n+1} \Rightarrow u_1z_1u_2z_2u_3 \ldots u_nz_nu_{n+1}$$

if K contains $X_1\mathbf{active}\,(Y_1)\,X_2 \ldots X_n\mathbf{active}(Y_n)\,X_{n+1}$ such that

for $i = 1, \ldots, n$, $y_i \in Y_i$ and $z_i \in \mathit{ContinuosRewriting}(y_i)$
for $i = 1, \ldots, n + 1$, $u_i \in X_i$

As usual, the *language of* G, $L(G)$, is defined as

$$L(G) = \{ w: S \Rightarrow^{*} w \text{ and } w \in T^{*} \},$$

where $\Rightarrow^{*}$ denotes the transitive and reflexive closure of $\Rightarrow$.

Example

Consider a multicontinuous grammar, G, with the following three productions

$$S \rightarrow bcb,\ b \rightarrow bb,\ c \rightarrow cc,$$

and these three selectors

$$\textbf{active}(\{S\}),\ \textbf{active}(\{b\}^{+})\{c\}^{*}\textbf{active}(\{b\}^{+}), \text{ and } \textbf{active}(\{b\}^{+})\textbf{active}(\{c\}^{+})\textbf{active}(\{b\}^{+})$$

For instance, G derives *bbbbccbbbb* as follows

$$S \Rightarrow bcb \ \Rightarrow\ bbcbb \Rightarrow bbbbccbbbb$$

Observe that G generates the following non-ontext-free language

$$L(G) = \{ b^{j}c^{k}b^{j} : j = 2^{i} \text{ and } k = 2^{r} \text{ so that } i \geq r \geq 0\}$$

Results

In the thesis, Section 4.2 discusses multicontinuous grammars. First, Section 4.2.1 conceptualizes them. Then, Section 4.2.2 proves that the six-nonterminal multicontinuous grammars characterize ***RE***.

5. Conclusion and Open Problems

First, this concluding chapter recalls the subject of investigation and its significance. Then, it summarizes the crucial results achieved in this thesis and points out several open problems closely related to these results.

The thesis investigates context-free multirewriting, which simultaneously applies context-free-like productions to several symbols during a single derivation step. This investigation is focused on the reduction of the number of nonterminals in the context-free multirewriting. Most results achieve this reduction without any decreasement of the generative power and, thereby, make the context-free multirewriting more economical. As obvious, this economization is highly desirable whenever multirewriting grammars are examined or applied.

The following two types of multirewriting grammars fulfill a key role in this thesis:

A. scattered context grammars
B. multisequential and multicontinuous grammars

(A) The scattered context grammars are based on sequences of context-free productions that simultaneously rewrites several nonterminals during a derivation step. (B) The multisequential and multicontinuous grammars use context-free-like productions that have a terminal or a nonterminal on their left-hand sides. By using extremely simple regular languages, called selectors, these grammars specify sequences of symbols rewritten during a derivation step and, in addition, place slight restrictions on the context appearing between the rewritten symbols; otherwise, they work by analogy with context-free grammars.

Results and Open Problems

Scattered Rewriting

Chapter 3, which consists of four sections, discusses scattered context grammars. Its first section gives an introduction to these grammars while the remaining three sections investigate them. The results of this investigation are summed up next.

Section 3.2 discusses the generative power of scattered context grammars. First, by a surprisingly simple proof, Section 3.2.1 demonstrates that these grammars characterize ***RE***. Then, by using a more complicated proof, Section 3.2.2 establishes this characterization based on the scattered context grammars containing no more than four nonterminals.

NOTE: After completing the thesis, the author proved that even the three-nonterminal scattered context grammars characterize ***RE*** (see [Meduna00b]).

Section 3.2.3 studies the economy of transformation of phrase structure grammars to scattered context grammars. More precisely, it describes a transformation that converts any phrase-structure grammar, *H*, to an equivalent scattered context grammar, *G*, so that G has no more than five more nonterminals than H.

Open Problem

Can the Section 3.2.3 transformation be improved so that G has only *i* more nonterminals than H, for some $i \in \{1, ..., 4\}$?

Section 3.3 discusses scattered rewriting based on terminating productions. It narrows its attention to the scattered context grammars whose sentential forms contain sequences of nonterminals formed by shuffling the left-hand sides of terminating productions. It proves that these grammars cannot even generate some languages in ***CS***, so they are significantly less powerful than ordinary scattered context grammars, discussed in Section 3.2. From this decreasement of the generative power, Section 3.2 derives that the one-nonterminal scattered context grammars do not generate some languages in ***CS***, so they do not characterize ***RE***. This statement and the above conjecture gives rise to the following question.

Open Problem

Do the two-nonterminal scattered context grammars characterize ***RE***?

Section 3.4 deals with the scattered context grammars without erasing productions, defined in Section 3.1. These grammars obviously cannot characterize ***RE*** because the language family that they generate is contained in ***CS*** (see [9]).

Open Problem

Is ***CS*** a proper superset of the family of the languages generated by the scattered context grammars without erasing productions?

Section 3.4 introduces some simple language operation and characterizes ***RE*** based on the introduced operations over the family of languages generated by the scattered context grammars without erasing productions. In fact, all the achieved characterizations are based on these grammars containing no more than five nonterminals.

Open Problem

Can the Section 3.4 characterizations be based on the *i*-nonterminal scattered context grammars without erasing productions, for some $i \in \{1, ..., 4\}$?

Multisequential and Multicontinuous Rewriting

Section 4.1 discusses multisequential grammars. It demonstrates that the six-nonterminal multisequential grammars characterize ***RE***.

Open Problem

Do the *i*-nonterminal multicontinuous grammars characterize ***RE***, for some $i \in \{1, ..., 5\}$?

Section 4.2 discusses multicontinuous grammars. It demonstrates that the six-nonterminal multicontinuous grammars characterize ***RE***.

Open Problem

Do the *i*-nonterminal multicontinuous grammars characterize ***RE***, for some $i \in \{1, ..., 7\}$?

Open Problem Area

Besides the above specific open problems, there exists a more general open problem area, which consists in a discussion of the subject of this thesis in terms of some other multirewriting grammars and related models (see [1-4], [6-8], [10], [12-39], [42-50]).

Appendix: Non-Context-Free Rewriting with a Reduced Number of Nonterminals

The appendix of the thesis briefly discusses some well-known grammars that rewrite their sentential forms in a non-context-free way. As before, it focuses this discussion on the reduction of the number of nonterminals in these grammars.

More specifically, it studies the *sequential context-dependent rewriting* performed by phrase-structure grammars. It characterizes ***RE*** by these grammars with no more than three nonterminals. Then, it investigates the *parallel context-dependent rewriting* performed by *EIL* systems. By analogy with the previous characterization, it proves that ***RE*** is defined by *EIL* systems containing no more than three nonterminals.

References

[1] Beigel, R. and Floyd, R. W.: *The Language of Machines*; Freeman, New York, (1994) ISBN 0-7167-8266-9.

[2] Csuhaj-Varju, E., Dassow, J., Kelemen, J., and Gh. Paun, Gh.: *Grammar Systems: A Grammatical Approach to Distribution and Cooperation*, Gordon and Breach, London, 1994 ISBN 803-9-80266-1.

[3] Csuhaj-Varju, E., Dassow, J., Kelemen, J., and Gh. Paun, Gh.: Eco-Grammar Systems: A grammatical framework for life-like interactions, *Artificial Life* 3 (1996), 27-38.

[4] Dassow, J. and Paun, Gh. : *Regulated Rewriting in Formal Language Theory*, Springer, Berlin, 1989 ISBN 3-05-500408-6.

[5] Gonczarowski, J. and Warmuth, M.K.: Scattered and Context-Sensitive Rewriting, *Acta Informatica* 20 (1989), 391-411.

[6] Habel, A. : Hyperedge Replacement: Grammars and Languages, *LNCS* 643, Springer, Berlin, 1992.

[7] Hein, J. L.: *Discrete Structures, Logic, and Computability*. Jones and Bartlett, London, (1995) ISBN 0-86720-477-x.

[8] Fernau, H.: Scattered Context Grammars with Regulation, *Analele Univ. Bucuresti: Matematica-Informatica*, XLV, 1 (1996).

[9] Greibach, S. and Hopcroft, J.E.: Scatterd Context Grammars, *J. Comp. Syst. Sci.* 3 (1969), 233-247.

[10] M. Ito (ed.): *Words, Languages, and Combinatorics*, World Scientific, Singapore,1992 ISBN 872-5-10675-1.

[11] Kleijn, H. C. M. and Rozenberg, G.: Multi Grammars, *Inter. J. Comput. Math* 12

(1983), 177-201.

[12] Mayer, O.: Some restrictive Devices for Context-Free Grammars, *Information and Control* 20 (1972), 69-92.

[13] Meduna, A. : A Note on Exponential Density of ETOL Languages, *Kybernetika* 22 (1986), 514-518.

[14] Meduna, A. : Evaluated Grammars, *Acta Cybernetika* 8 (1987), 169-176.

[15] Meduna, A. : Characterization of the Chomsky Hierarchy through Sequential-Parallel Grammars, *Rostock. Math. Kolloq.* 32 (1987), 4 - 14.

[16] Meduna, A. and Horvath, G. : On State Grammars, *Acta Cybernetica* 8 (1988), 237 - 245.

[17] Meduna, A. : Context Free Derivations on Word Monoids, *Acta Informatica* 27, 781 - 786, 1990.

[18] Meduna, A. : Generalized Forbidding Grammars, *International Journal of Computer Mathematics* 36 (1990), 31 - 38.

[19] Meduna, A. : Global Context Conditional Grammars, *J. Inform. Process. Cybern.* 27 (1991), 159 - 165.

[20] Meduna, A. : Symbiotic E0L Systems , *Acta Cybernetika* 12 (1992), 164 -172.

[21] Meduna, A. : A Formalization of Sequential, Parallel, and Continuous Rewriting, *International Journal of Computer Mathematics* 39 (1993), 24 -32.

[22] Meduna, A. : Canonical Scattered Rewriting, *International Journal of Computer Mathematics* 51 (1993), 122 - 129.

[23] Meduna, A. and Csuhaj-Varju, E.: Grammars with Context Conditions, *EATCS Bulletin* 32 (1993), 112 - 124.

[24] Meduna, A. : Matrix Grammars under Leftmost and Rightmost Restrictions, in vol. *Mathematical Linguistics and Related Topics* (Gh. Paun, ed.), The Publ. House of the Romanian Academy, Bucharest, (1994), 243 - 257.

[25] Meduna, A., Crooks, C., and Sarek, M.: Syntactic Complexity of Regulated Rewriting, *Kybernetika* 30(1994), 177 - 186.

[26] Meduna, A., and Gopalaratnam, M. : On Semi-Conditional Grammars with Productions Having either Forbidding or Permitting Conditions, *Acta Cybernetica* 11 (1994), 309 - 323.

[27] Meduna, A. : Syntactic Complexity of Scattered Context Grammars, *Acta Informatica* 32 (1995), 285 - 298.

[28] Meduna, A. : A Trivial Method of Characterizing the Family of Recursively Enumerable Languages by Scattered Context Grammars, *EATCS Bulletin* 56 (1995), 104 - 106.

[29] Meduna, A. : Syntactic Complexity of Context-Free Grammars over Word Monoids, *Acta Informatica* 33 (1996), 457 - 462.

[30] Meduna, A. : Four-Nonterminal Scattered Context Grammars Characterize the Family of Recursively Enumerable Languages, *International Journal of Computer Mathematics* 63 (1997), 67-83.

[31] Meduna, A. : On the Number of Nonterminals in Matrix Grammars with Leftmost Derivations, *LNCS* 1217 (1997), 27 - 38.

[32] Meduna, A. : Six-Nonterminal Multi-Sequential Grammars Characterize the Family of Recursively Enumerable Languages, *International Journal of Computer Mathematics* 65, 1997, 179 -189.

[33] Meduna, A. : Economical Transformation of Phrase-Structure Grammars to Scattered Context Grammars, *Acta Cybernetica* 13, 1998, 225 - 242.

[34] Meduna, A. : Uniform Rewriting Based on Permutations, *International Journal of Computer Mathematics* 69, 1998, 57 - 74.

[35] Meduna, A. : Descriptional Complexity of Multi-Continues Grammars, *Acta Cybernetica* 14, 1998, 375-384.

[36] Meduna, A. : Prefix Pushdown Automata, *International Journal of Computer Mathematics* 71, Gordon and Breach, London, 1999, s. 215-228.

[37] Meduna, A. : Terminating Left-Hand Sides of Scattered Context Productions, *Theoretical Computer Science*, North-Holland, Amsterdam, 1999, (in press).

[38] Meduna, A. : Generative Power of Three-Nonterminal Scattered Context Grammars, *Theoretical Computer Science*, North-Holland, Amsterdam, (in press).

[39] Meduna, A. : *Automata and Languages: Theory and Applications*, Springer, London, (2000) ISBN 1-85233-074-0.

[40] Meduna, A. and Kolar, D.: Descriptional Complexity of Parallel Grammars with Respect to the Number of Nonterminals, In: *Words, Sequences, Languages: Where Computer Science Meets,* Biology, and Linguistics (Carlos Martin-Vide, ed.), Kluwer, New York (in press)

[41] Milgram, D. and Azriel Rosenfeld, A.: A Note on Scattered Context Grammars, *Information Processing Letters*, 1(2), 1971,47-50.

[42] Paun, Gh.: On simple Matrix Languages versus Scattered Context Languages, *RAIRO* 16 (1982), 245-253.

[43] Paun , Gh.: *Artificial Life: Grammatical Models*, Black Sea University Press, Bucharest, Romania (1995) ISBN 973-0-00131-6.

[44] Paun, Gh.: *Mathematical Linguistics and Related Topics*, The Publ. House of the Romanian Academy, Bucharest, Romania (1995) ISBN 502-0-10675-1.

[45] Rozenberg, G. : Selective Substitution Grammars (Towards a Framework for Rewriting Systems), Part I: Definitions and Examples, *J. Inform. Process. Cybern.* 13 (1977), 455-

463.

[46] Rozenberg, G., and Salomaa, A.: *The Mathematical Theory of L Systems*, Academic Press, New York, (1980) ISBN 0-12-597140-0.

[47] Rozenberg, G. and Salomaa, A. (eds.): *Handbook of Formal Languages*, Volume 1 through 3, Springer, (1997) ISBN 3-540-60649-1.

[48] Salomaa, A., *Computation and Automata*, Cambridge University Press, Cambridge, England, (1985) ISBN 0-521-30245-5.

[49] H. J. Shyr, *Free Monoids and Languages*, Hon Min Book Comp., Taichung, 1991.

[50] Ullman, J. D.: Computational Aspects of VLSI, *Computer Science Press*, Rockville, Maryland, (1984) ISBN 3-523-00233-1.

[51] Virkkunen, V.: On Scattered Context Grammars, *Acta Universitatis Ouluensis*, Series A, Mathematica 6 (1973), 75-82.

Souhrn

Bezkontextové gramatiky aplikuji jedno pravidlo během každého derivačního kroku, a tedy přepisují řetězce sekvenčně. Na rozdíl od tohoto klasického sekvenčního způsobu přepisování, předkládané habilitační práce diskutuje gramatiky, které současně aplikují několik bezkontextových pravidel během jediného derivačního kroku (*context-free multirewriting*) a tím vlastně tvoří gramatický model paralelního způsobu výpočtu, který dnes představuje jedno z centrálních témat informatiky. Kromě tohoto fundamentálního významu máji tyto gramatiky celou řadu dalších předností. K těm nejpodstatnějším patří zvýšení generativní síly a možnost redukce některých gramatických komponent jako např. počtu neterminálů. Výzkum těchto vlastností tvoří jádro presentované habilitační práce.

Práce se soustředí na studium vlastnosti následujících dvou typu gramatik:

(A) gramatiky s rozptýleným kontextem
(B) gramatiky s paralelními selektory

(A) **Gramatiky s rozptýleným kontextem** (*scattered context grammars*) jsou založené na sekvencích bezkontextových pravidel, které simultánně přepisují několik neterminálů během jediného derivačního kroku. Habilitační práce ukazuje, že gramatiky s rozptýleným kontextem charakterizuji třídu jazyků typu 0. Práce dokonce dokazuje, že této charakterizace lze dosáhnout i pro gramatiky s rozptýleným kontextem, které obsahuji nejvíce čtyři neterminály. Kromě tohoto zásadního výsledku, práce studuje radu podmínek, které ovlivňují sílu gramatik s rozptýleným kontextem. Konkrétněji, habilitační práce demonstruje, ze redukce počtu neterminálů na jediný neterminál či eliminace zkracujících pravidel oslabuje silu těchto gramatik. Na druhé straně však ukazuje, že prostřednictvím některých jednoduchých jazykových operací, jako např. slabé identity, lze získat charakterizaci celé třídy jazyků typu 0 prostřednictvím gramatik s rozptýleným kontextem i když v nich existenci zkracujících pravidel vyloučíme.

(B) **Gramatiky s paralelními selektory symbolů** (*multisequential grammars*) jsou založeny na jednoduchých regulárních selektorech. Tyto selektory specifikuji symboly, které mohou byt simultánně přepsány během jednoho derivačního kroku. Práce ukazuje, ze celou třídu jazyků typu 0 lze charakterizovat prostřednictvím těchto gramatik s nejvíce šesti neterminály.

Gramatiky s paralelními selektory podřetězců (*multicontinuous grammars*) jsou rovněž založeny na regulárních selektorech. V tomto případě však selektory specifikují celé podřetězce, které jsou současně přepisované. Práce dokazuje, že třídu jazyků typu 0 lze charakterizovat prostřednictvím těchto gramatik s nejvíce šesti neterminály.

Závěr práce podtrhuje význam dosažených výsledků a diskutuje řadu otevřených problémů a domněnek, včetně domněnky, že třídu jazyků typu 0 lze charakterizovat i prostřednictvím gramatik s rozptýleným kontextem, které obsahují nejvíce tři neterminály. Krátce po dokončení habilitační práce autor tuto domněnku rigorózně dokázal v [38].