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**WAVELET ANALYSIS FOR SIGNAL DETECTION -
APPLICATIONS TO EXPERIMENTAL
CARDIOLOGY RESEARCH**

**VLNKOVÁ ANALÝZA PRO DETEKCI SIGNÁLŮ –
APLIKACE V EXPERIMENTÁLNÍM
KARDIOLOGICKÉM VÝZKUMU**

SHORT VERSION OF HABILITATION THESIS



Brno 2002

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vlnková analýza, spojitá vlnková transformace, detekce signálů, kardiologie

KEY WORDS

wavelet analysis, continuous wavelet transform, signal detection, cardiology

MÍSTO ULOŽENÍ PRÁCE

archiv FEKT VUT v Brně

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1 INTRODUCTION

Today's signal processing tools have been developed through dozens of years to cover all possible application areas - from data analysis to data compression. However, some specific but everyday technical problems cannot be effectively resolved without techniques that are more complex. One of the newest additions has been wavelets.

Although the subject area of wavelets has developed mostly over the last fifteen years, it is connected to older ideas in many other fields including pure and applied mathematics, physics, computer science, and engineering. Roots of "modern" wavelet theory have been founded at several places in the late 1970's and in the 1980's. First, J. Morlet came up with an alternative for the short-time Fourier transform. Morlet followed two aims: to gain high time-resolution for high frequency transients and good frequency resolution for low frequency components. While these two goals form a trade-off in traditional short-time Fourier transform, Morlet decided to generate the transform functions in a different way: he took a windowed cosine wave (using a smooth window) and compressed it in time to obtain higher frequency function, or spread it out to obtain lower frequency function. In order to examine time changes, these functions were shifted in time as well.

The **first aim** of this thesis is to describe tools of wavelet analysis with emphasis to continuous-time wavelet transform. Thus, comprehensive overview on published theory and practical results is presented. **Second aim** is to present details of features of wavelet-based tools and present possibilities of their application. **Third aim** is to show how the wavelet tools can be implemented in experimental cardiology research.

In the thesis, the term wavelet analysis represents expansion of a discrete-time or continuous-time signal on wavelet bases. Generally said, the expansion can be provided by any well-known signal processing tool such as Fourier transform. However, wavelet analysis exploits a simple but genius idea - the signal is expanded on a set of dilated or compressed functions

$$\psi\left(\frac{t-b}{a}\right).$$

The dilation a - the scale - is the key factor that allows to change both time and frequency resolution when analyzing the signal. The signal can be analyzed to detect short-time frequency-limited events, it can be more effectively compressed, or noise present in the signal can be suppressed in time- and frequency-selective manner.

Wavelets have been extensively used in biomedical engineering since their first formal formulation in the end of 1980's. First special journal issue¹ on wavelets in

¹ IEEE Engineering in Medicine and Biology Magazine 14(2), 1995 - Special Issue on Time-Frequency and Wavelet Analysis.

biomedical engineering was released in March 1995. The guest editor M. Akay chose outstanding papers on analysis of uterine EMG signals, EEG signals in epilepsy, heart sound signals, ECG signals in ventricular fibrillation, fetal breathing rate, respiratory-related evoked potentials, and others. Those papers showed wavelets in applications where classical analysis tools failed.

1.1 Tools of time-frequency analysis

Traditional frequency analysis by Fourier transform has many alternative methods. The necessity of their application is provoked by non-stationary characteristics of the signals being analyzed. Most published methods are short-time Fourier transform, Wigner-Ville distribution, and wavelet transform.

Short-time Fourier transform is locally applied Fourier transform. The signal $f(t)$ is first multiplied by a shifted window function $w(t-\tau)$. Then, the conventional Fourier transform is taken. The resulted transform is represented by a two-dimensional function. A number of various window functions have been proposed to achieve good-time frequency resolution. A good example is the use of Gaussian window proposed by Gabor in 1946.

Wigner distribution or more often Wigner-Ville distribution is a well-known example of expansion. Its intention is to get estimation of instantaneous power spectrum. The attractive feature of Wigner-Ville distribution is the possible high time-frequency resolution. For signals with a single time-frequency component the Wigner-Ville distribution gives a clear and concentrated energy ridge in the time-frequency plane. However, in the case of multicomponent signals, cross terms and interferences appear.

Continuous wavelet transform (CWT) uses shifts and scales (dilation and contraction) of the prototype function $\psi(t)$ instead of its shifts and modulations. CWT gives conical pattern showing good frequency resolution for high scales corresponding to low frequencies and poor frequency resolution for low scales corresponding to high frequencies.

1.2 Time and frequency resolution

In any signal processing applications, time-frequency localization - i.e. localization of a given basis function in time and frequency - is an important consideration. Signal domain methods require a high degree of localization in time while frequency domain methods demand a high degree of localization in frequency. This results in trade-off that can be optimized but not made ideal.

Definition of localization of a basis function is usually based on how it covers certain area in time-frequency plane. The elementary area in the plane is called a tile. Ideally, the tile is represented by a small rectangular window centered on the place of interest in the time-frequency plane.

To center the tile in the plane, the transforms used for time-frequency representation use elementary operations such as shift in time, modulation in frequency, and scaling. Obviously, shift in time by τ results in shifting of the tile by τ across the time axis. Similarly, modulation by $e^{i\omega_s t}$ shifts the tile by ω_s . All elementary operations conserve the area of the tile. In addition, note that the tile shape is never ideally rectangular or never has infinitely narrow dimensions. Its real shape is determined by basis functions used for expansion.

Consider a signal $f(t)$ centered on t_0 with its frequency spectrum $F(\omega)$ centered on ω_0 . Let us define time resolution as time width Δ_t of $f(t)$ by its root mean square spread and frequency resolution as frequency width Δ_ω of $F(\omega)$ by square of the 2nd moment of $|F(\omega)|^2$

$$\Delta_t^2 = \frac{1}{E} \int_{-\infty}^{\infty} (t - t_0)^2 |f(t)|^2 dt, \quad (1.1)$$

$$\Delta_\omega^2 = \frac{1}{E} \frac{1}{2\pi} \int_{-\infty}^{\infty} (\omega - \omega_0)^2 |F(\omega)|^2 d\omega, \quad (1.2)$$

where E is energy of the signal. Resolutions in time and frequency are related in uncertainty principle [6]. The principle sets a bound on the maximum theoretical joint resolution in time and frequency represented by a product $\Delta_t \Delta_f$. If $f(t)$ decays faster than $1/\sqrt{t}$ as $t \rightarrow \infty$, then uncertainty principle asserts [19]

$$\Delta_t^2 \Delta_\omega^2 \geq \frac{1}{2}. \quad (1.3)$$

2 MULTIREOLUTION AND WAVELET TRANSFORM

Wavelets arose in diverse scientific areas from mathematics to engineering and have been described in various ways. They have been formalized in terms of multiresolution analysis and continuous wavelet transform, and further in discrete wavelet transform and subband coding.

Multiresolution theory has been formulated in 1986 by S. Mallat [13] and Y. Meyer. It provides a framework for understanding and description of wavelet bases. The basic idea of the analysis is based on existence of a sequence of embedded approximation spaces. If some six requirements that define the multiresolution analysis are satisfied, then there exists an orthonormal basis on which a signal can be expanded. Such expansion is non redundant, effective, and easily understandable. In fact, its interpretation is not more difficult than interpretation of Fourier analysis. Further, multiresolution analysis can be described in terms of subband coding using multirate filter banks.

Continuous wavelet transform represents different approach but possesses further desired features. Multiresolution theory - well described and ready to be imple-

mented - "only" allows (orthogonal) dyadic expansion that corresponds to signal processing by an octave-band filter bank. Thus, frequency and time resolution is preset and unchangeable. Opposite to it, continuous wavelet transform decomposes a signal with arbitrary resolution in both time and frequency. Thus, CWT has received significant attention in its ability to zoom in on singularities, which has made it an attractive tool in the analysis of non-stationary and fractal signals. CWT can be discretized to obtain arbitrarily sampled time-frequency plane of wavelet coefficients.

2.1 Multiresolution analysis

Multiresolution theory based on multiresolution analysis and other mathematical tools gives a foundation for exact description of expansion on orthogonal bases in a Hilbert space $\mathbf{L}_2(\mathbf{R})$. Let us use definition adopted by Daubechies in [6]: a multiresolution analysis consists of a sequence of embedded closed subspaces $\cdots \mathbf{V}_2 \subset \mathbf{V}_1 \subset \mathbf{V}_0 \subset \mathbf{V}_{-1} \subset \mathbf{V}_{-2} \cdots$ such that they posses the following properties: upward completeness, downward completeness, scale invariance, shift invariance, and existence of an orthonormal basis $\{\varphi(t-n) | n \in \mathbf{Z}\}$, for \mathbf{V}_0 , where $\varphi \in \mathbf{V}_0$. The function $\varphi(t)$ is called the scaling function.

The scaling function composes an orthogonal basis for \mathbf{V}_0 . Using the scale invariance and the shift invariance, we can write orthonormal basis $\{\varphi_{m,n}\}$, $n \in \mathbf{Z}$ for the space \mathbf{V}_m [6]

$$\varphi_{m,n}(t) = 2^{-m/2} \varphi(2^{-m}t - n), \text{ for } m, n \in \mathbf{Z}. \quad (2.1)$$

Daubechies also proves [6] existence of an orthonormal basis

$$\psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m}t - n), \text{ for } m, n \in \mathbf{Z}, \quad (2.2)$$

such that $\{\psi_{m,n}\}$, $n \in \mathbf{Z}$, is an orthonormal basis for a space \mathbf{W}_m , where \mathbf{W}_m is the orthogonal complement of the space \mathbf{V}_m in \mathbf{V}_{m-1} . The function $\psi(t)$ is called the wavelet function.

2.2 Continuous wavelet transform

Let us consider redundant representation of continuous-time functions in terms of two variables - scale and shift. The representation is called continuous wavelet transform. Although the CWT is redundant, it is worth of use because of its interesting features. Consider a family of functions obtained by shifting and scaling a wavelet function $\psi(t)$ such as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi^* \left(\frac{t-b}{a} \right), \quad (2.3)$$

where $a, b \in \mathbf{R}$, $a > 0$, and a star denotes complex conjugate. The normalization ensures that $\|\psi_{a,b}(t)\| = \|\psi(t)\|$. The wavelet function has to oscillate. This, together with the decay property, has given $\psi(t)$ the name wavelet or "small wave".

The continuous wavelet transform of a function $f(t)$ is defined as (e.g. [6])

$$CWT(a, b) = \int_{-\infty}^{\infty} \psi_{a,b}(t) f(t) dt. \quad (2.4)$$

The function $f(t)$ can be recovered from its wavelet transform by the reconstruction formula

$$f(t) = \frac{1}{C_{\psi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{a^2} CWT(a, b) \psi_{a,b}(t) da db. \quad (2.5)$$

The CWT possesses properties similar to properties of other linear transformations: linearity, shift property, localization property, energy conservation, and scaling property.

Because of high redundancy in continuous $CWT(a, b)$, it is possible to discretize the transform parameters. CWT often uses a hyperbolic grid in case of linear grid of a . A special case of the hyperbolic grid is a dyadic grid when scales are powers of 2. That is, the time-scale plane (a, b) is discretized as

$$a = a_0^m, \quad b = n b_0 a_0^m, \quad (2.6)$$

where $m, n \in \mathbf{Z}$, $a_0 > 1$, $b_0 > 0$. In this manner, large basis functions (when is a_0^m large) are shifted in large steps, while small basis functions are shifted in small steps. In order for the sampling of the time-scale plane to be sufficiently fine, a_0 has to be chosen close to 1, and b_0 close to 0. The discretized family of wavelets is now

$$\psi_{m,n}(t) = a_0^{-m/2} \psi(a_0^{-m}t - nb_0). \quad (2.7)$$

2.3 Wavelets for time-frequency localization

Wavelets are basis functions used for expansion. They are characterized by a number of properties that determine their use in the frame of time-frequency localization. Formally, a real-valued function $\psi(t)$ is called a wavelet if it satisfies two constraints defined by

$$\int_{-\infty}^{\infty} \psi(t) dt = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \psi^2(t) dt = 1. \quad (2.8)$$

The first part of Eq. 2.8 states that the wavelet oscillates, the second part says the wavelet must be nonzero somewhere. The properties of wavelets may serve as a key for selection of function for a specific application. Briefly, while analysis needs even non-orthogonal wavelets, compression requires orthogonal and smooth wavelets. Filtering may require symmetrical functions and rational coefficients of filters corresponding to wavelets. The following properties are most discussed in literature: orthogonality, compact (finite) support, rational coefficients of corresponding filters, symmetry, smoothness, and analytic expression.

2.3.1 Real-valued wavelets

The most used and/or discussed real-valued wavelets are: Haar wavelet, family of Daubechies wavelets, Morlet wavelet, Meyer wavelet, Mexican hat wavelet, family of Coiflet wavelets, family of Symlet wavelets, and biorthogonal wavelets. Time and frequency resolution of various wavelets differ. The ideal resolution value is represented by an equality curve $\Delta_t^2 \Delta_\omega^2 = 0.5$. Results for all wavelets lay right and above the equality curve. The closer to the equality curve, the better time resolution, frequency resolution, or both resolutions are. The results for selected wavelets are summarized in Tab. 2.1.

| wavelet | Δ_t^2 | Δ_ω^2 | $\Delta_t^2 \Delta_\omega^2$ |
|-----------------|--------------|-------------------|------------------------------|
| Morlet | 0.7071 | 0.7081 | 0.5007 |
| Gaussian No.2 | 0.7637 | 0.6889 | 0.5261 |
| Meyer | 0.8418 | 0.9824 | 0.8271 |
| Daubechies No.2 | 1.540 | 9.424 | 14.51 |
| Haar | 0.5775 | 130.6 | 75.44 |

Tab. 2.1 Time resolution, frequency resolution, and time-frequency resolution of selected real-valued wavelets. Theoretical minimum of $\Delta_t^2 \Delta_\omega^2$ is 0.5.

2.3.2 Complex-valued wavelets

The most used and/or discussed complex-valued wavelets are: Complex Gaussian wavelets, Complex Daubechies wavelets, Complex Kingsbury wavelet, Complex Morlet wavelets, Complex Frequency B-spline wavelets, Complex Shannon wavelets. Time and frequency resolution of various complex-valued wavelets differ too. The results for selected wavelets are summarized in Tab. 2.2.

| wavelet | Δ_t^2 | Δ_ω^2 | $\Delta_t^2 \Delta_\omega^2$ |
|--|--------------|-------------------|------------------------------|
| cpx Morlet No.1-0.5 (real or imaginary part) | 0.3533 | 1.416 | 0.5006 |
| cpx Kingsbury (real part) | 1.552 | 3.418 | 5.306 |
| cpx Kingsbury (imaginary part) | 1.565 | 3.681 | 5.765 |
| cpx Daubechies No.6 (real part) | 2.526 | 3.066 | 7.749 |
| cpx Daubechies No.6 (imaginary part) | 2.647 | 3.363 | 8.904 |

Tab. 2.2 Time resolution, frequency resolution, and time-frequency resolution of selected complex-valued wavelets. Theoretical minimum of $\Delta_t^2 \Delta_\omega^2$ is 0.5.

3 WAVELET ANALYSIS FOR SIGNAL DETECTION

Wavelet analysis of signals is modern approach to solution of many digital signal processing problems nowadays. When proper wavelet tools are chosen and properties of the tools met conditions based on certain signal parameters, the wavelets can powerfully serve to reach significantly "better" results compared to e.g. results of Fourier transform methods.

Wavelet tools and all their variants result in 2-dimensional continuous or discretized output of two parameters: scale and time shift. While time-shift corresponds to time axis of the signal to be analyzed, scale corresponds, but is not equal, to frequency. This makes interpretation of wavelet analysis less explicit than in the Fourier analysis. In practice, wavelet analysis output is often called time-frequency image or time-frequency spectrum. However, correct interpretation has to consider frequency spectra of individual scaled wavelets.

"How-to" of wavelet signal analysis naturally comes out of the transform properties. Localization property, shift property, and scaling property compose the fundamentals of all detection algorithms based on the wavelet transform. However, when using wavelets, we are faced several problems that must be resolved before the signal is analyzed.

First of all, we have to choose the following parameters of the wavelet analysis: type of the wavelet transform, type of the mother wavelet, and scales. All the three wavelet analysis parameters have several or even infinite number of options. The parameters are closely related and must be considered at the same time.

The type of the wavelet transform is usually the first step. It leads to the general decision whether to use orthogonal expansion or overcomplete expansion. Orthogonal expansion usually leads to a bank of octave filters representing dyadic discrete-time wavelet transform. Overcomplete expansion is usually represented by the continuous-time wavelet transform computed on a given grid to discretize the resulted continuous-time function. The wavelet type should be set according to the signal being analyzed. The general selection of wavelets is based on the shape of the wavelet in time domain, its length (support), and smoothness. The last step is based on how many details in what frequency range is needed. This is done by selection of scales for what the analysis will be computed. The simplest case is the dyadic discrete-time wavelet transform. Here, the scales are set to powers of two. The selection is reduced to "how deep" the analysis will be done. In other words, we have to select the number of the coarsest levels. Overcomplete CWT also allows us to choose arbitrary scales according to actual signal properties, if known.

3.1 Detection of waves using CWT

Detection of waves and short-time events is an important part of signal analysis. Thus, the signal can be examined to find differences from a reference signal, track

long-term trends, and multiple time overlapping and/or frequency overlapping changes. Traditional time-domain and frequency-domain detection methods are based on correlation and cross-correlation, coherence, cross-spectra, cepstra, and many other signal processing tools. Time-frequency approach exploits expansion on series to decompose the signal into multiple frequency bands. Further, time and frequency resolution can be individually changed in the bands and thus the analysis algorithm can be adapted to the signal being detected.

3.1.1 Detection using envelope contour

The user is faced various problems when CWT should be used to detect and bound time events in wavelet signal analysis. The problems result from the nature of the time-frequency image that is too complex for direct analysis. Originally, the output of the CWT is two-dimensional and may be depicted by 3D plot, 2D shaded image, 2D contour image, and 1D plot of cross-sections through the output along time axis or scale axis. A number of parameters can be observed in the images/plots: presence and position of the peaks, slope of the peaks in various directions, etc.

The signal waves can be more precisely detected using envelope contours. First, a contoured image of the output is taken. Further, square root of absolute value of the output is taken to visualize more details by increasing the image dynamics. The image is sliced at eight levels regularly spaced between zero and maximum of the output function. The contour image $C_L(a, b)$ for set of M levels L is defined as

$$C_L(a, b) = \begin{cases} 1 & \text{if } \text{sqrt}(\text{abs}[CWT(a, b)]) \in \langle L_k - \varepsilon; L_k + \varepsilon \rangle, \\ 0 & \text{if } \text{otherwise} \end{cases}, \quad (3.1)$$

for all k 's, where $k=1..M$ is a level number, L_k is a k -th level, ε is a small number. The levels are defined as

$$L_k = \frac{\max_{a \in A, b \in B} [CWT(a, b)]}{M} * k, \quad (3.2)$$

where A is a set (or an interval) of all considered scales, and B is a set of all signal samples (or a time interval) of the analyzed signal. Second, only that part of the contour L_1 , which is the closest to the highest frequency, is considered. Such a contour is called an envelope contour EC and is defined as

$$EC(b) = \min_{a \in A, C_{L_1}(a, b) \neq 0} [a], \quad (3.3)$$

for all b 's. The envelope contour EC is represented by a 1D signal that can be analyzed by common detectors or recognizers.

3.1.2 Basics of complex-valued CWT analysis

Complex-valued wavelet transform plays a special role in signal analysis. Complex nature of wavelets provides further improvement in signal detection compared to

real-valued wavelet analysis. This is possible by using so called dual-tree processing [10] through cross-correlation with real and imaginary parts of wavelets. The resulted complex-valued time-frequency image can be further analyzed by detection of significant attributes in its modulus and phase. In this way, not only the waves can be detected but also various shapes of the waves can be distinguished.

Let us present the complex-valued CWT abilities on an example. The tested signal has been artificially corrupted by a short-time event - a discontinuity in first derivative at $t=84$ msec. Such discontinuity can hardly be seen in the time domain without further processing.

Then, the signal has been transformed using the complex Morlet wavelet No. 1-0.5. As the CWT promises to detect short-time events regardless their frequency contents, we should obtain a significant pattern in a resulting time-frequency image. Studying the modulus CWT output depicted in Fig. 3.1 (b), one can easily find a narrow object (islet) located at scales $1/a=0.2-0.3$ (mid-frequencies). Although the islet is low in value, it is detectable with relatively good time resolution.

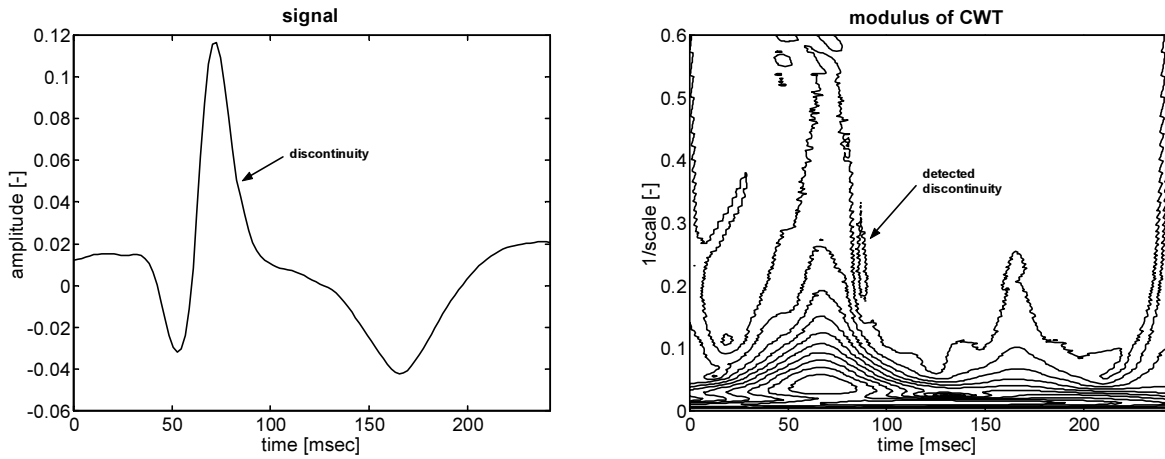


Fig. 3.1 (a) Signal with artificial discontinuity in first derivative at $t=84$ msec, (b) modulus of CWT using complex Morlet wavelet No.1-0.5.

Complex-valued CWT analysis may be further improved by considering its phase. The phase of the complex-valued CWT of signals has characteristic structure. It contains phase discontinuities along time axis that reveals as vertical line objects. The lines correspond to extrema and inflection points of waves. The discontinuity in the signal is displayed as an additional line in the phase.

3.1.3 Detection of waves using modulus of complex-valued CWT

To study complex-valued CWT behavior on various signal shapes, let us consider a first derivative of square modulus and phase. Thus, we may wish to detect extrema and inflection points that clearly define particular waves of the signal. By approximating the derivative of square modulus of CWT, we can establish connections between local extrema of $|CWT(a,b)|^2$ and inflection points and local extrema of a signal being analysed. $|CWT(a,b)|^2$ is considered as a function of time shift b

here (with constant scale a). Local maxima (minima) of $|CWT(a, b)|^2$ always represent inflection points (local minima or maxima) of the signal, respectively.

One can find that the maxima and minima partly overlay each other. To detect extremal points or segments in the CWT, the following simple method can be used. The method is based on detection of local maxima of the CWT modulus. A local maximum of the CWT modulus located at time b_2 at scale a is defined as

$$|CWT(a, b_1)| < |CWT(a, b_2)| > |CWT(a, b_3)|, \quad (3.4)$$

where $b_1 < b_2 < b_3$, $b_1 \rightarrow b_2$ and $b_3 \rightarrow b_2$ (b_1, b_2, b_3 are adjoining samples when the signal being analysed is a discrete-time signal). If inequality in Eq. 3.4 is applied on all a 's, adjoining $\{a, b_2\}$ pairs compose separated "lines" called maximum curves. The maximum curves coincide with the minor extrema of the function.

Further analysis of the modulus of the CWT is possible by detection of minima. A local minimum of the CWT modulus located at time b_2 at scale a is defined similar to Eq. 3.4 with reversing the inequalities. If inequality equation is applied on all a 's, adjoining $\{a, b_2\}$ pairs compose separated "lines" called minimum curves.

3.1.4 Detection of waves using phase of complex-valued CWT

To study behavior of CWT of signals with local extrema or inflection points, we consider the case of functions exhibiting local symmetry (or anti-symmetry) properties. $CWT(a, b_0)$ of locally symmetric $f(t)$ is real and its phase is then 0 or π . $CWT(a, b_0)$ of locally anti-symmetric $f(t)$ is imaginary and its phase is then $\pi/2$ or $-\pi/2$.

For analysis purposes, the phase can be thresholded to obtain a less complex output image. Thresholded phase is defined as

$$P_{th}(a, b) = \begin{cases} 1 & \text{for } |\arg[CWT(a, b)]| \in \langle th - \varepsilon, th + \varepsilon \rangle \\ 0 & \text{for } |\arg[CWT(a, b)]| \notin \langle th - \varepsilon, th + \varepsilon \rangle \end{cases}, \quad (3.5)$$

where $th = \{-\pi/2, 0, \pi/2, \pi\}$ is threshold, ε is a small real number. The phase thresholding at $\{-\pi/2, 0, \pi/2, \pi\}$ results in binary image with vertical "lines" located at positions of phase steps.

3.2 Conclusions

Behavior of the complex-valued continuous wavelet transform (CWT) as response to various signal types has been discussed. Experiments have shown that CWT may serve as a detector of signal changes. We have proved that the complex-valued wavelet transform can be used for detection of waves represented by local maxima and minima of the signal. Further, it can be used for recognition of symmetric and antisymmetric waves. As the first step of the complex-valued wavelet analysis in detection, square modulus (or simply modulus) of CWT should be computed. Thus, maxima, minima, or even inflection points are found as detectable maxima or min-

ima. Phase of CWT is used to distinct between extrema and inflection points in the signal being analysed.

However, the examples discussed above represent ideal conditions that may be far from reality. First, all signals are recorded with nonzero signal to noise ratio. The noise may produce local extrema that disturb CWT and make the detection more complicated. Further, no signal is exactly symmetric (antisymmetric). Even small asymmetry degrades resulting peaks in CWT modulus.

Consider a repetitive signal of a series of waves. After a certain time delay, the signal is corrupted by additive noise represented by short-time waves of lower value than the signal amplitude. The noise partially overlaps the signal in time and frequency domain. Comparing CWT of both original and corrupted signals, one can find detectable differences in modulus as well as phase of CWT. The modulus reveals the differences as additional peaks in its image. Although the differences are small in value, they change shape of original peaks in modulus image or they generate separated peaks. The phase responses even more sensitively regardless the noise wave amplitude. Any new signal component is revealed as a new phase step along time axis.

Analysis of signals using complex-valued continuous wavelet transform is the first step to detect possible changes or alternans. In the second step, modulus and phase must be thoroughly examined. However, the complex-valued time-frequency image is too complex. Its further analysis may fail when using simple methods (position of main peaks, number of new peaks in modulus, number of new π -steps in phase, etc).

4 WAVELET ANALYSIS IN CARDIOLOGY RESEARCH

Wavelet analysis has been linked to signal processing in early 1990's. A number of applications to various fields have been described since that time. A typical examples are telecommunication, mechanical engineering, geology, climatology, oceanology, astrophysics, computer science, and biomedical engineering.

Signal detection applications of wavelets in biomedical engineering include broad class of tasks [4]. In cardiology research, wavelet analysis is exploited in electrocardiographic (ECG) signal compression, ventricular arrhythmia analysis, heart rate variability analysis, cardiac pattern characterization, late potentials analysis, fetal ECG extraction, heart sounds analysis, ventricular pressure variability, high-resolution ECG analysis, detection of T-wave changes and ST-T complex changes, detection of conduction block, and many others.

Wavelets are an efficient tool for analysis of short-time changes in signal morphology. As pointed out by Unser and Aldroubi in [18], the preferred type of wavelet transform for signal analysis is the redundant one that is the continuous wavelet transform in opposition to the non-redundant type corresponding to the expansion on orthogonal bases (multiresolution analysis). The reason is that the CWT

allows decomposition on an arbitrary scale. Thus, frequency bands of interest can be studied properly at chosen resolution.

In the following text, wavelet analysis is applied to cardiology. There, electrocardiographic signals are used to detect a pathological process in the heart (myocardial ischemia). ECG signals are recorded from the same subject in two phases: control (physiological signal) and after the event (artery occlusion causing acute ischemia). Thus, the original signal and corrupted signal are taken and can be used to design an effective detection algorithm. However, even the physiological signal is noisy itself. This is caused by dynamic nature of the signal source and the dynamic systems between the source and recording electrodes. The analysed ECG signal is then composed of repetitions that vary (beat-to-beat variations). Such variations may negatively influence the detection and may increase a number of false positive events (low specificity). The same negative effect may be caused by other unwanted noise, e.g. powerline noise, moving artifacts, myopotentials, etc.

4.1 Detection of myocardial ischemia

In the Western world, sudden cardiac death remains a leading cause of death. In the majority of the cases, sudden death is caused by lethal arrhythmia's preceded by acute myocardial ischemia. The study of ischemic heart disease can reveal mechanisms of its genesis and its influence in the electrophysiology of the heart. Results of the study could then contribute to a better understanding and both pre-infarction and post-infarction treatment of the disease. Further, the results could help to develop a new noninvasive method needed in cardiology diagnostics [7].

Here, wavelets are used to detect acute myocardial ischemia caused by occlusion of a coronary artery. This application may help to understand fundamentals of electrophysiological changes underlying myocardial ischemia.

4.1.1 Electrophysiological manifestation of acute myocardial ischemia

Coronary ischemia is characterized by interrelated metabolic ionic and neurohumoral events that alter membrane properties of cardiac cells, causing electrophysiological changes. A deteriorated myocardial perfusion causes a potential difference between ischemic and normal regions during the ST segment. While ST-depression is considered the most common manifestation of exercise-induced cardiac ischemia, ST-elevation may be related to severe posterior, subepicardial or transmural ischemia, or with myocardial infarction. Sometimes, fixed ST elevation or depression may occur. ST changes may therefore be ambiguous and are not always able to reflect changes in myocardial perfusion [8]. Concluding, cardiac ischemia may result in changes in one or more leads of the electrocardiogram. The electrocardiographic criterion for detecting ischemia is ST-segment displacement. The value of this criterion for predicting coronary artery disease is limited and is reported between 47 and 91% for sensitivity and between 69 and 97% for specificity.

Since ischemia causes conduction changes, irregular depolarization (activation) of the myocardium may occur. This would be manifested as intra-QRS changes. There is much evidence that ischemia changes in the heart muscle may cause alterations in the QRS spectrum, as an expression of the fragmentation of ventricular depolarization. Abboud [1] detected high-frequency changes in signal-averaged QRS complexes of dogs and human patients caused by ischemia. Further, focal reduction of high-frequency components of the QRS complex under myocardial ischemia induced by percutaneous transluminal coronary angioplasty have been showed [8]. Therefore, a technique similar to the spectrotemporal analysis of late potentials [12] might prove useful in early detection of ischemic changes. This idea is further promoted by Petterson [15], where ST-segment analysis criteria are combined with root-mean-square values of QRS high-frequency components criteria.

4.1.2 t-test analysis of signal changes

The main issue in analysis of QRS changes is to localize the abnormal time-scale components contained in the ECG signal to identify a given cardiac disease. A method compares the representations of the ECGs of the studied (ischemic) sample to a reference (control) sample. The significant abnormality mapping is assessed by comparing the mean value of each of the wavelet transforms of the two studied populations by means of an two-way two-tailed t-test to test for the null hypothesis that means are equal. If the null hypothesis is rejected, the statistically significant appearance of QRS changes in time-frequency spectra is confirmed.

Data were collected under the following protocol. The signals of one-minute length were recorded in 15 time instances from three orthogonal leads (X, Y, Z). Thus, a record from control period (0 min) and six records from ischemic period (1, 3, 5, 10, 15, 20 min) were taken. A typical signal recorded from X-lead is depicted in Fig. 4.1. One can see several time-domain changes during all periods. Comparing to control period, ST-segment elevates in ischemic period. Further, T-wave increase in amplitude and shifts to QRS-complex during ischemic period. Other minor changes are visible too.

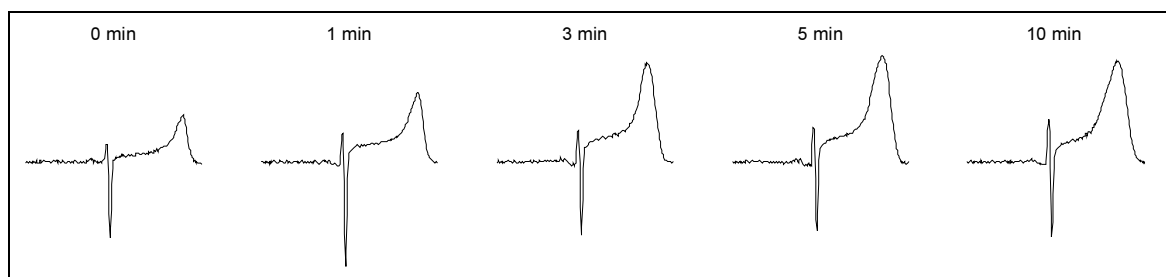


Fig. 4.1 Typical recordings from myocardial ischemia experiment. Legend: 0 min - baseline recording, 1 min to 10 min - acute ischemia.

The data were reduced to set null hypothesis for statistical t-test. Two recordings were chosen to generate studied sample and reference sample. Studied sample were

composed of recordings after 3 minutes of ischemia, the reference sample were composed of recordings from control period. L consequent heart cycles were chosen in each recording of all n experiments. Heart cycles are physiologically of (slightly) different length. Therefore, M samples centered on a fiducial point FP were chosen. FP was set as an arithmetic center between QRS-onset and QRS-offset of each heart cycle. Thus, $n*L*2$ recordings of heart cycles were taken for the test. Overall number of signal samples for statistical analysis was $n*L*2*M$. The results in this chapter are shown for $n=15$, $L=10$, $M=250$ (i.e. 500 msec for sampling rate of 500 Hz).

Time-frequency manifestation of myocardial ischemia should be graphically presented before some preprocessing and statistical analysis of wavelet data will be done. Such presentation should prove that statistical analysis will likely reject null hypothesis and that time-frequency image bear significant information on electrophysiological changes due to acute myocardial ischemia.

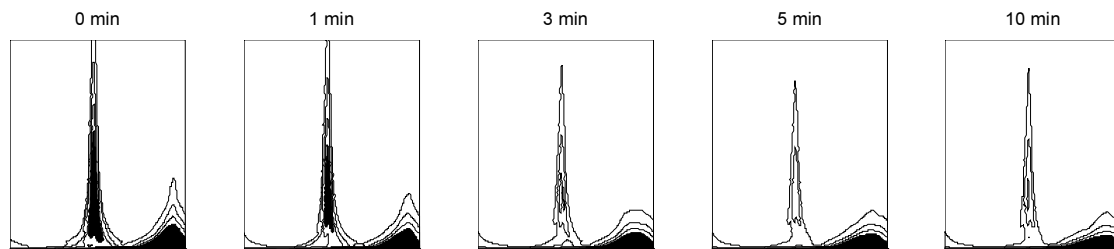


Fig. 4.2 Continuous wavelet transform of recordings from Fig. 4.1 using Kingsbury wavelet. 0 min - baseline recording, 1 min to 10 min - acute ischemia.

Fig. 4.2 shows CWT of all recordings from Fig. 4.1. Time-frequency images reveal some changes within QRS complexes during ischemia (center part of particular pictures). Further, energy dissipation within QRS complex and energy accumulation in T-wave is obvious.

It should be pointed out that QRS complex shape and duration may physiologically vary. These variations are small but may significantly influence statistical test. Therefore, some filtering method should be applied. An efficient algorithm uses median filtering which computes median QRS complex from a set of L consequent QRS complexes. Each of $n*2$ median QRS complexes were transformed using CWT. Thus, $n*2$ matrices of $M \times S$ wavelet coefficients were computed. S is a number of scales of CWT. These matrices provided input data for two-tailed t-test.

The CWT analysis discussed above using statistical results is likely a good ischemic marker. Sufficient efficiency was achieved for recordings from X-lead and in CWT modulus ($p < 0.001$ for larger areas within QRS complexes in time-frequency images). The results can be used to develop an automatic on-line detector of acute myocardial ischemia. The detector would be based on comparison of a current time-frequency image to time-frequency image computed from a signal recorded at the beginning of diagnostic procedure.

4.2 Hidden Markov model based detector of acute myocardial ischemia

Another detector uses two discrete density hidden Markov models. First model was trained on a set of CWT images of control QRS complexes. Second model was trained on a set of CWT images of QRS complexes from ischemic period. Both models were build as ten-state left-right structures.

The left-right property of the model was chosen to follow time-nature of an ECG signal where samples of the signal follow each other in one direction. The number of states was experimentally chosen as usual in Markov model applications.

Individual states were represented by index of attribute vectors. Attribute vectors are simply generated by taking vectors of M wavelet coefficients for each time instant. Indexes of attribute vectors were set according to a codebook, which consisted of minimized number of representative attribute vectors generated during training phase.

The detector based on the above hidden Markov model was tested on a set of signals from 11 experiments where three vectorcardiograms were recorded before LAD occlusion (control period) and after 3 minutes of acute ischemia (ischemic period). One QRS complex of each cardiogram was included into analysis. The size of the codebook was 50 attribute vectors. Number of states of the model was experimentally set up to ten. Six models representing non-ischemic and ischemic state in vectorcardiograms from lead X, Y, and Z were built up. Each signal was transformed by CWT using Morlet wavelet and processed by two corresponding models. The results for lead X are shown in the Tab. 4.1. The first row in the Tab. 4.1 represents that 81.8% control signals recorded from X-lead were recognized as control signals. The second row represents that 90.9% ischemic signals recorded from X-lead were recognized as ischemic signals.

| lead | tested signal | used model | score [%] |
|------|---------------|------------|-----------|
| X | control | control | 81.8 |
| | ischemic | ischemic | 90.9 |

Tab. 4.1 Results of myocardial ischemia detection using ten-state hidden Markov model.

5 WAVELET ANALYSIS IN EDUCATION

Wavelet analysis has undergone significant growth in the past few years, with many successes in the efficient analysis, processing, and compression of signals and images. It is based on the idea of frequency-scale decomposition of signals and images, which offers many advantages over the traditional frequency decompositions. Unfortunately, the new technique requires broad mathematical framework and adequate knowledge of digital signal processing techniques. Thus, a comprehensive course

that includes fundamentals of multiresolution analysis, continuous and discrete-time wavelet transform with appropriate examples is needed for potential users.

Inclusion of the topic into the university teaching system is possible when sufficient fundamentals of signal processing and Fourier theory are given. A good example is a set of two consequent one-semester courses "Signals and Systems" and "Digital Signal Processing". Then, a new course of a given topic can follow either in graduate or post-graduate study. The objectives of the course are as follows: i) to give an introduction to wavelet analysis in one and two dimensions, and ii) show that it may be practically used in applications of filtering and compression.

Wavelet analysis has already been partially included into teaching process at Department of Biomedical Engineering, BUT. First, multirate signal processing, half-band filters, filter banks, and discrete-time wavelet transform are covered in an optional course "New Methods of Signal Processing" supervised and lectured by Dr. Jiří Kozumplík, computer laboratory exercises led by Dr. Ivo Provazník. Second, a number of semestral projects on wavelet analysis have been supervised by Dr. Ivo Provazník. Although there is limited time for solving such projects during regular semester, students are able to learn necessary wavelet basics. Very good knowledge of digital signal processing including Fourier theory is necessary. Most of the semestral projects have been continued and completed as Master's theses in 1996-2001.

Third, a chapter on wavelet transform has been included into second edition of the book [9] by J. Jan: Digital Signal Filtering, Analysis and Restoration and the basic principles of wavelet transform are presently taught in the graduate course "Digital Signal Processing".

6 CONCLUSIONS

Wavelets have generated an enormous interest in both theoretical and applied science areas, especially over the past ten years. New advancements in the science are occurring at such a rate that even the meaning of the term "wavelet analysis" continuously keeps changing to incorporate all new ideas. In the thesis, wavelet analysis is discussed in terms of multiresolution analysis as a framework for orthogonal expansion of functions and series.

The text summarizes fundamentals of wavelet theory: basic mathematical background is given for basic understanding of time-frequency representation, common time-frequency analysis tools are presented such as wavelet transform and short-time Fourier transform, and time-frequency resolution is defined in frame of basis function of time-frequency decomposition. Further, three roots of time-frequency analysis are discussed: multiresolution analysis, subband coding and filter banks, and continuous wavelet transform (CWT). Concerning the wavelet transform, a comprehensive overview of important wavelets. Both real-valued and complex-valued wavelets are discussed.

The thesis presents a novel contribution to wavelet-based detection of signals using complex-valued wavelets. The first technique exploits a localization property. A simple technique generates an envelope contour of narrow signal elements and thus the CWT analysis result into one-dimensional domain. The envelope contour is suitable to be used by common detectors of extrema. The second technique is based on analysis of CWT modulus that provides information on local extrema and inflection points of the analysed signal. The technique uses local maxima of the CWT modulus and tracks CWT modulus ridges. Thus, so-called maximum curves are generated. Their position and length may serve for further detection. Another technique is based on analysis of CWT phase that provides further information on inflection points of the analysed signal. Using a thresholding algorithm, curves of break phases are generated. These curves may complete maximum curves for more effective detection.

The study is completed by examples of two applications of CWT in cardiology research. The first example shows how modulus of complex-valued CWT can be used in detection of myocardial ischemia. A novel algorithm employing a hidden Markov model working on time-scale image of recorded ECG signals is described. Further, the model finds electrophysiological intra-QRS changes, which is a novel technique compared to classical ST-segment analysis. The second example shows how modulus of complex-valued CWT can be used in detection of electrophysiological changes in the heart caused by neurological drugs. A simple algorithm tracking two highest peaks in the time-scale image is discussed.

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Souhrn

Algoritmy zpracování signálů byly vyvíjeny po několik posledních desetiletí pro řadu aplikačních oblastí - od předzpracování přes analýzu signálů až po kompresi dat. Některé specifické, ale časté problémy nemohou být efektivně řešeny bez nových sofistikovanějších technik. Jednou z nejnovějších významných metod, které přispívají do této oblasti, jsou vlnky (orig. wavelets).

Prvním cílem habilitační práce je podat náhled na problematiku vlnkové analýzy s důrazem na spojitou vlnkovou transformaci (CWT). V práci je prezentován ucelený popis publikované teorie s uvedením praktických výsledků. **Druhým cílem** práce je detailně popsat vlastnosti nástrojů vlnkové analýzy a naznačit možnosti jejich praktického využití. **Třetím cílem** je uvedení praktických implementací metod vlnkové analýzy v experimentálním kardiologickém výzkumu.

Pojem vlnková analýza v práci reprezentuje rozvoj signálů na bázi vlnek. Rozvoj může být proveden např. Fourierovou transformací. Vlnková analýza ale využívá jednoduché, avšak účinné myšlenky - transformačním jádrem je dilatovaná mateřská vlnková funkce. Faktorem dilatace je tzv. škála, která určuje časové i frekvenční rozlišení při analýze signálu. Takto mohou být v signálech detekovány krátkodobé úzkopásmové jevy, signály mohou být efektivně komprimovány, nestacionární šum může být potlačen s časovou i frekvenční selekcí, atd.

Teze obsahují nové příspěvky k vlnkové detekci signálů s použitím komplexních vlnkových funkcí. První metoda využívá lokalizační vlastnosti CWT. Jednoduchý algoritmus generuje tzv. konturovou obálku elementů analyzovaného signálu a výsledek časově-frekvenční analýzy tak převádí do jednorozměrné oblasti. Konturová obálka je snadno zpracovatelná běžnými detektory extrémů. Druhá metoda je založena na analýze modulu časově-frekvenčního spektra. Modul CWT poskytuje spolehlivou informaci o lokálních extrémech a inflexních bodech analyzovaného signálu. Popisovaná metoda využívá lokálních maxim CWT. Takto jsou generovány tzv. "maximální křivky", jejichž pozice a délka slouží pro detekci. Poslední popisovaná metoda je založena na analýze fáze CWT, která poskytuje další informace o vlastnostech signálu v inflexních bodech. Prahováním fáze CWT signálu jsou generovány tzv. křivky fázových zlomů, které přispívají ke zvýšení efektivnosti detekce.

Habilitační práce je doplněna podrobně popsány příklady dvou aplikací spojitě vlnkové transformace v kardiologickém výzkumu. První příklad ukazuje, jak je modul komplexní CWT použit k detekci myokardiální ischemie. Nový algoritmus využívá skrytý Markovův model pracující s časově-frekvenčním obrazem zaznamenaného EKG signálu. Model detekuje elektrofyziologické změny projevující se uvnitř QRS komplexu, což představuje nový přístup v porovnání s tradiční analýzou ST segmentu. Druhý příklad ukazuje použití analýzy modulu komplexní CWT pro detekci elektrofyziologických změn v srdci způsobených vlivem neurologických léků. Zde je popsán jednoduchý algoritmus založený na sledování dvou nejvyšších vrcholů časově-frekvenčního obrazu.