Parallel Computations Based on Modified Numerical Integration Methods

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Abstract. Even though the idea of parallel computing and parallel connection of high amount of microprocessors is attractive, it is not easy to reach big increase in performance compared to single processor approach. The potential of parallel data processing has already been studied. It was found, that even a small percentage of sequential steps may lead to high reduction of performance of the entire system. This is the consequence of the fact, that most algorithms were not developed for heavy parallel systems.

Keywords: Modified Numerical Integration Methods, Parallel Computations
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THE IDEA OF PARALLEL COMPUTATIONS

This paper concentrates on large systems of parallel microprocessors. The idea of this approach comes from analogue methods of computations. Analogue methods are basically parallel methods and their analysing shows that independent parallel cooperation of multiple processors may be implemented by applying differential calculus. Using analogue methods with mechanization of the equations, independent parallel cooperation of microprocessors is going to be effective if each microprocessor is going to be numerically integrating.

Mechanization of the equations

All we shall assume here is that analog components exist in order to carry out the requisite mathematical operations. The equations to be considered are

$$y'' = C, \quad y(0) = A, \quad y'(0) = 0, \tag{1}$$
$$y'' + k \cdot y = C, \quad y(0) = A, \quad y'(0) = 0, \tag{2}$$

where C is a constant.

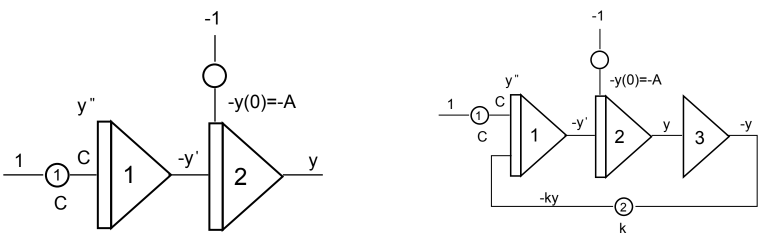

FIGURE 1. Mechanization of the equation (1) - left, mechanization of the equation (2) - right

We begin the mechanization of Eq. (1) (see Fig. 1 - left) by inserting unit across linear potentiometer 1 which is set at the value C. The output of this potentiometer is C, which is equal to y'', according to Eq. (1). The C is therefore the input to integrator 1 whose output is $-y'$ (recognizing that the conventional d-c amplifier produces a sing inversion). The term $-y'$, is, in turn, fed to integrator 2, the output of which is $y(t)$, which is the desired solution. This is an example of an "open-loop" mechanization.

Eq. (2) is an example of the more common type of problem, the analogue mechanization of which involves feedback (see Fig. 1 - right), see more in [1]. Typically in both examples the integrators 1 and 2 are working simultaneously (in parallel).

MODIFIED NUMERICAL INTEGRATION METHODS

The idea of analogue principles above is used in numerical integrators. The only standard operational amplifiers are replaced by parallel numerical integrators (microprocessors).

The algorithms of parallel cooperation of microprocessors can be derived from one step of numerical solutions. This parallel cooperation of independent microprocessors may be completed using arbitrary chosen numerical integration formula including Euler's method, 2nd, 3rd and 4th order Runge-Kutta method, Adams-Bashforth method etc.

Basic concept of modified numerical integration with brief description of one-step numerical integration methods, which is necessary for explaining of the entire concept of parallel cooperation of microprocessors is now presented.

A special kind of differential equations $y' = f(y)$ is analyzed.

Test Example 1

$$y' = -y, \qquad y(0) = 1. \tag{3}$$

The equations for calculation of the first step y_1 of differential equation (3) using modified numerical integration methods are shown in Table 1 for Euler method (E), Runge-Kutta methods (RK).

TABLE 1. Formulas for solving differential equation (3).

E	$y_0 = y(0)$	3rd order	$K_0Y = h(-y_0)$
	$K_0Y = h(-y_0)$		$K_1Y = h(-(y_0 + \frac{K_0Y}{2}))$
	$y_1 = y_0 + K_0Y$		$K_2Y = h(-(y_0 + 2K_1Y - K_0Y))$
R-K	$y_0 = y(0)$		$y_1 = y_0 + \frac{K_0Y}{6} + 2\frac{K_1Y}{3} + \frac{K_2Y}{6}$
2nd order	$K_0Y = h(-y_0)$	4th order	$K_0Y = h(-y_0)$
	$K_1Y = h(-(y_0 + K_0Y))$		$K_1Y = h(-(y_0 + \frac{K_0Y}{2}))$
	$y_1 = y_0 + \frac{K_0Y}{2} + \frac{K_1Y}{2}$		$K_2Y = h(-(y_0 + \frac{K_1Y}{2}))$
			$K_3Y = h(-(y_0 + K_2Y))$
			$y_1 = y_0 + \frac{K_0Y}{6} + \frac{K_1Y}{3} + \frac{K_2Y}{3} + \frac{K_3Y}{6}$

Similarly $y_2, y_3, \ldots, y_n$ are calculated.

Test Example 2

$$y' = z, \qquad y(0) = 1, \tag{4}$$

$$z' = -y, \qquad z(0) = 0. \tag{5}$$

The equations for calculation the first steps y_1, z_1 of differential equations (4),(5) using modified numerical integration methods are shown in Table 2.

The Table 2 gives an idea of a parallel computation of the solution y_1, z_1 again for Euler method (E), Runge-Kutta methods (RK). Similarly $y_2, z_2, y_3, z_3 \ldots, y_n, z_n$ are parallely calculated (mathematical operations for y_1, z_1 are independent and the same in corresponding lines of Table 2). A corresponding parallel cooperation is presented with respect to the mechanization of the equations in Fig. 2 - right.

TABLE 2. Formulas for solving differential equations (4),(5).

E	$y_0 = y(0)$	$z_0 = z(0)$
	$K_0Y_0 = h(z_0)$	$K_0Z_0 = h(-y_0)$
	$y_1 = y_0 + K_0Y_0$	$z_1 = z_0 + K_0Z_0$
R-K	$y_0 = y(0)$	$z_0 = z(0)$
2nd order	$K0Y = h(z_0)$	$K0Z = h(-y_0)$
	$K1Y = h(z_0 + K0Z)$	$K1Z = h(-(y_0 + K0Y))$
	$y_1 = y_0 + \frac{K0Y}{2} + \frac{K1Y}{2}$	$z_1 = z_0 + \frac{K0Z}{2} + \frac{K1Z}{2}$
4th order	$K0Y = h(z_0)$	$K0Z = h(-y_0)$
	$K1Y = h(z_0 + \frac{K0Z}{2})$	$K1Z = h(-(y_0 + \frac{K0Y}{2}))$
	$K2Y = h(z_0 + \frac{K1Z}{2})$	$K2Z = h(-(y_0 + \frac{K1Y}{2}))$
	$K3Y = h(z_0 + K2Z)$	$K3Z = h(-(y_0 + K2Y))$
	$y_1 = y_0 + \frac{K0Y}{6} + \frac{K1Y}{3} + \frac{K2Y}{3} + \frac{K3Y}{6}$	$z_1 = z_0 + \frac{K0Z}{6} + \frac{K1Z}{3} + \frac{K2Z}{3} + \frac{K3Z}{6}$

GENERAL DESCRIPTION OF CONTINUOUS SYSTEMS

Fig. 2 - right is an extremely simple example of a parallel cooperation of integrators in a continuous systems. Generally, initial problems described by autonomous systems of differential equations are in the form

$$
\begin{aligned}
w_1' &= f_1(w_1, w_2, \ldots, w_n, x_1, x_2, \ldots, x_n,), \quad w_1(0) = w_1^0, \\
w_2' &= f_2(w_1, w_2, \ldots, w_n, x_1, x_2, \ldots, x_n,), \quad w_2(0) = w_2^0, \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
w_n' &= f_n(w_1, w_2, \ldots, w_n, x_1, x_2, \ldots, x_n,), \quad w_n(0) = w_n^0, \\
x_1 &= g_1(w_1, w_2, \ldots, w_n, x_1, x_2, \ldots, x_n,), \\
x_2 &= g_2(w_1, w_2, \ldots, w_n, x_1, x_2, \ldots, x_n,), \\
&\vdots \\
x_n &= g_n(w_1, w_2, \ldots, w_n, x_1, x_2, \ldots, x_n,).
\end{aligned} \tag{6}
$$

and the corresponding block diagram is in Fig. 2 - left.

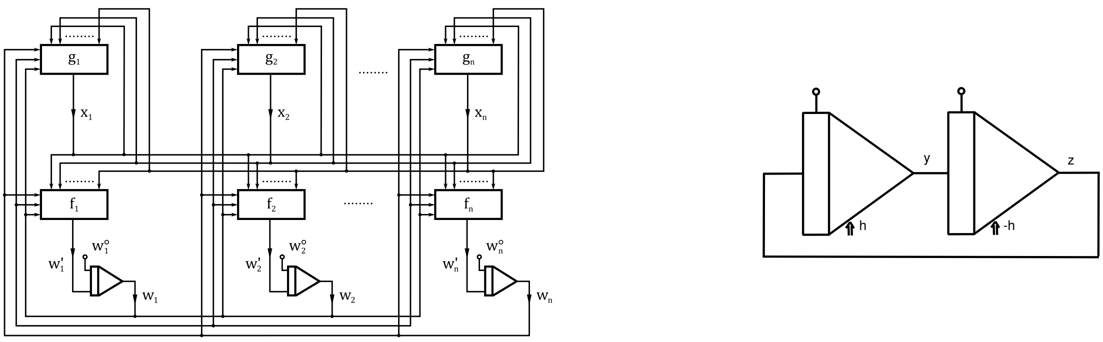

FIGURE 2. General description - left, inverting and noninverting solution - right

It is clear that integrators in Fig. 2 - left can work in parallel. The question now is how to calculate also functions $f_1, f_2, \ldots, f_n$ in parallel.

Many test examples have been completed to confirm that, if the functions on the right-hand sides of (6) are of a particular type frequently encountered in engineering applications, a sequence of substitutions can be found that transforms the original system into a new system with polynomials on the right-hand sides.

Test Example 3
Simple transformation

$$y' = y + \sin(t), \qquad y(0) = y_0, \tag{7}$$

is presented.

New initial problem

$$\begin{aligned} y' &= y + y_1, & y(0) &= y_0, & (8)\\ y_1' &= y_2, & y_1(0) &= 0, & (9)\\ y_2' &= -y_1, & y_2(0) &= 1, & (10) \end{aligned}$$

can be seen in Fig. 3 (right part).

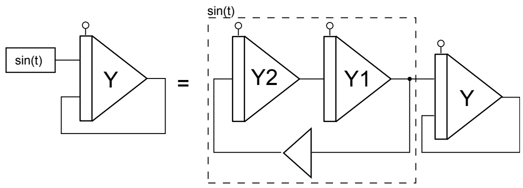

FIGURE 3. Corresponding block diagram

CONCLUSION

Systems of homogenous linear differential equations, electronic circuits simulations, control systems, partial differentials equations and systems of algebraic differential equations are typical applications of parallel cooperations of integrators.

We can often observe that, if the functions on the right-hand sides of (6) are of a particular type frequently encountered in engineering applications, a sequence of substitutions can be found that transforms the original system into a new system with polynomials on the right-hand sides.

The only integrators, multiplayer and dividers are used in our numerical integrators. There is no doubt that extremely precise and fast algorithms of Modern Taylor Series Method must be applied [2].

Detailed information will be given during the ICNAAM 2012 conference.

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