Running Large-Scale Ultrasound Simulations on 256 Salomon Nodes

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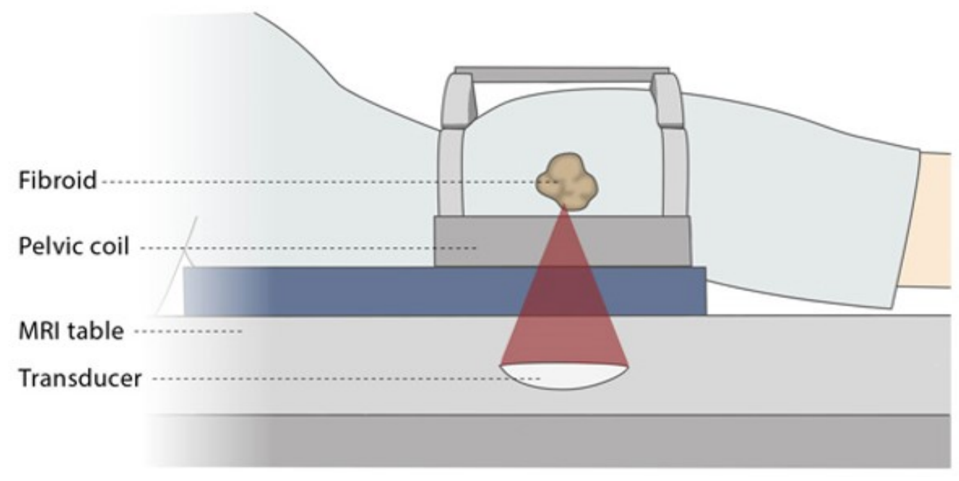
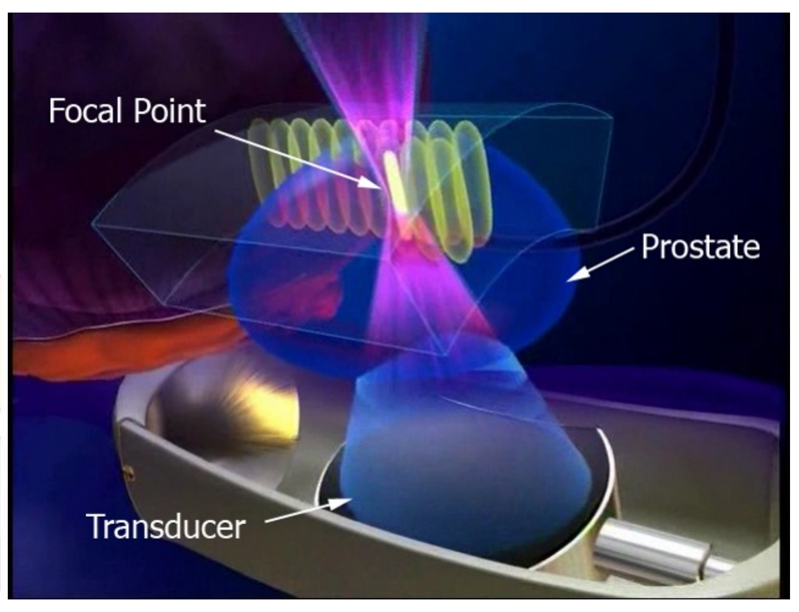
Overview

High-intensity focused ultrasound (HIFU) is an emerging non-invasive cancer therapy that uses tightly focused ultrasound waves to destroy tissue cells through localised heating. The treatment planning goal is to select the best transducer position and transmit parameters to accurately target the tumour. The path of the ultrasound waves can be predicted by solving acoustic equations based on mass, momentum and energy conservation. However, this is a computationally difficult problem because the domain size is very large compared to the acoustic wavelength.





Nonlinear Ultrasound Wave Propagation in Tissue

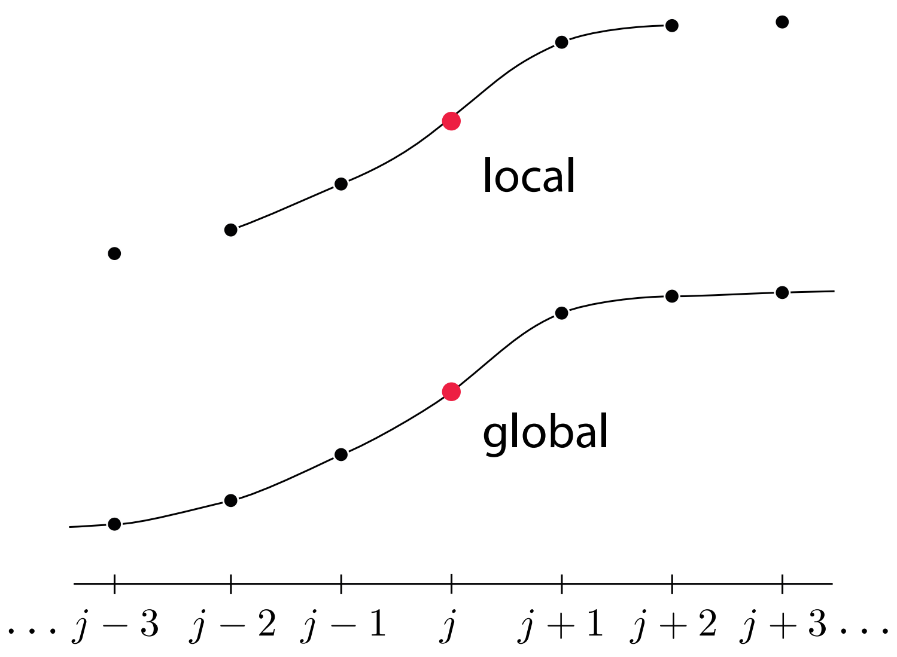
The governing equations must account for the nonlinear propagation of ultrasound waves in tissue, which is a heterogeneous and absorbing medium. Accurate acoustic absorption is critical for predicting ultrasound dose under different conditions. The required acoustic equations can be written as:

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_0}\nabla p + \mathbf{S}_F \qquad \text{(momentum conservation)}$$

$$\frac{\partial \rho}{\partial t} = -(2\rho + \rho_0)\nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \rho_0 + S_M \qquad \text{(mass conservation)}$$

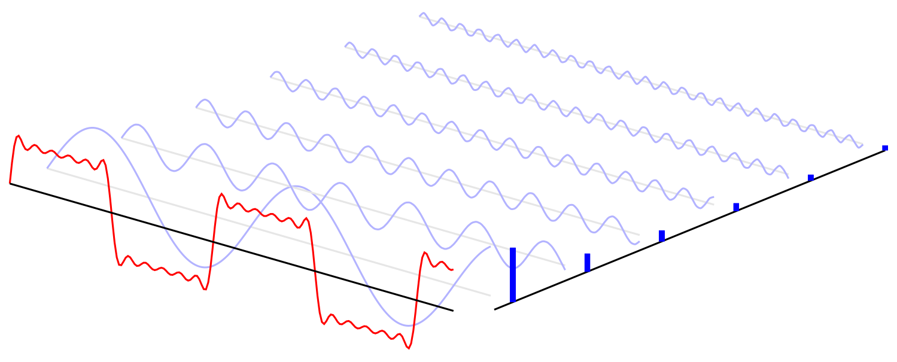
$$p = c_0^2\left(\rho + \mathbf{d} \cdot \nabla \rho_0 + \frac{B}{2A}\frac{\rho^2}{\rho_0} - L\rho\right) \qquad \text{(pressure-density relation)}$$

These equations are discretized using the k-space pseudo-spectral method and solved iteratively. This reduces the number of required grid points per wavelength by an order of magnitude compared to finite element or finite difference methods. For uniform Cartesian grids, the gradients can be calculated using the fast Fourier transform (FFT).



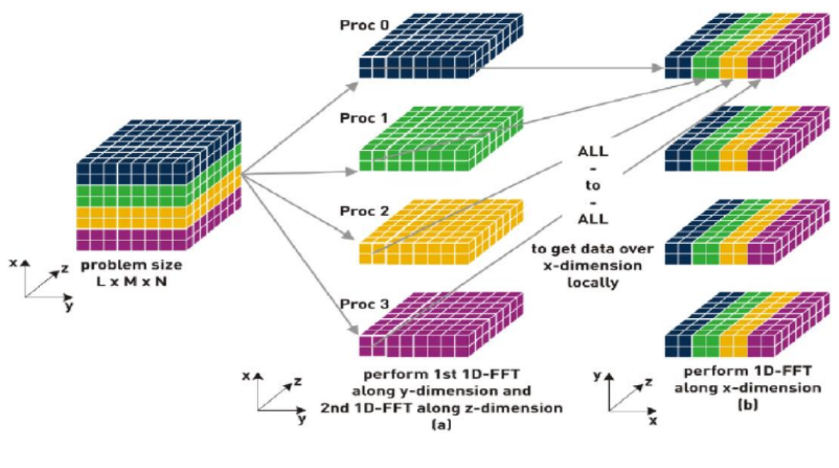
$$\frac{\partial f}{\partial x} = \frac{\frac{1}{12}f^{j-2} - \frac{2}{3}f^{j-1} + \frac{2}{3}f^{j+1} - \frac{1}{12}f^{j+2}}{\Delta x}$$

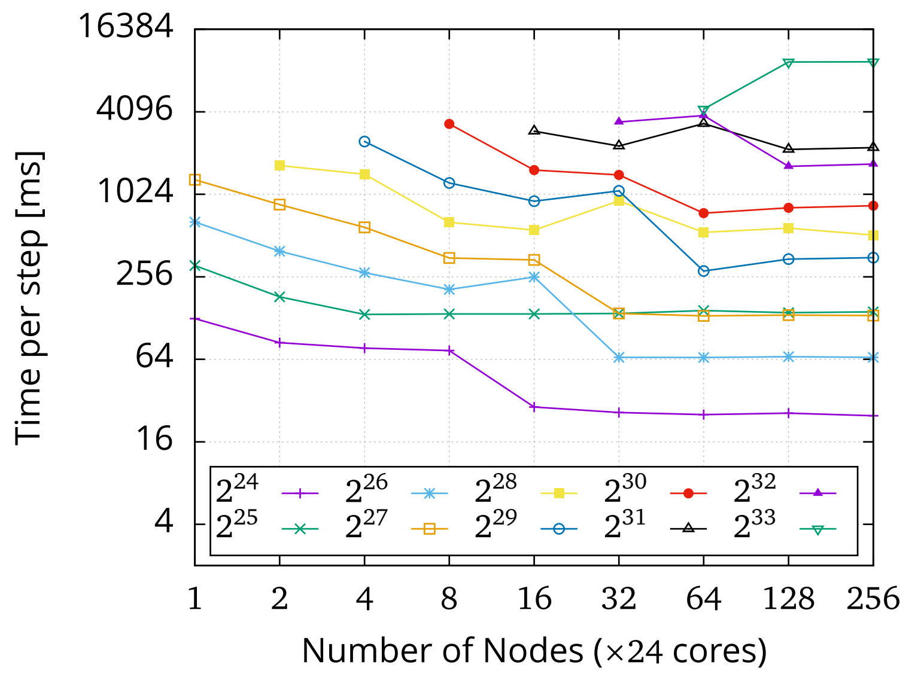
$$\frac{\partial f}{\partial x} = \mathscr{F}^{-1}\left\{ik_x \mathscr{F}\left\{f\right\}\right\}$$



Global Fourier Basis on Cluster

Evaluation of derivatives requires computation of 3D fast Fourier transform which involves global transposition of the matrix which in turn requires multiple All-to-All communications. The global communication significantly limits scaling of the simulation code.

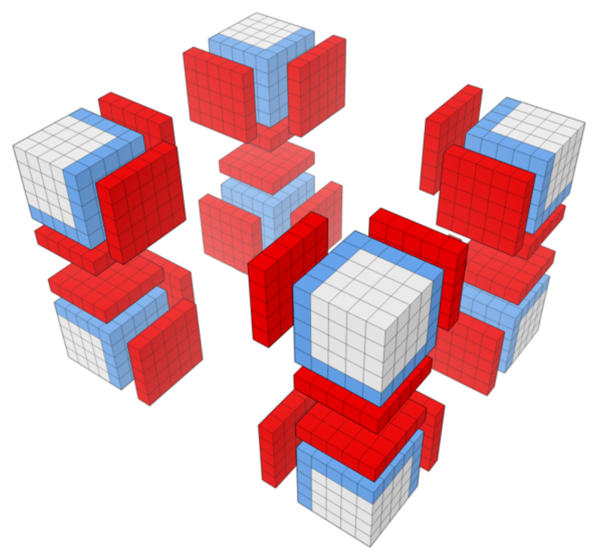


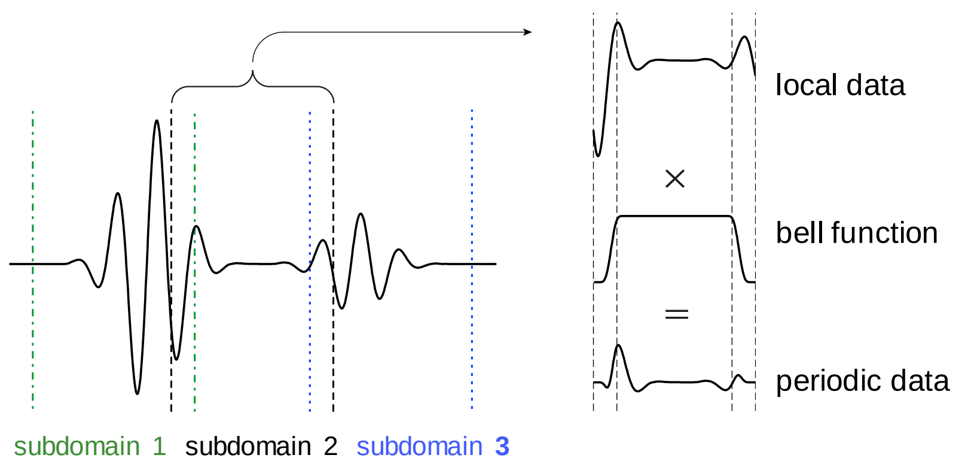


Local Fourier Basis Decomposition

Local domain decomposition reduces the communication burden by partitioning the domain into a grid of local subdomains where gradients are calculated locally and the global communication is replaced by the nearest-neighbor overlap exchange. The gradient calculation with the overlap on an i-th subdomain reads as follows (b is a bell function smoothening the subdomain interface):

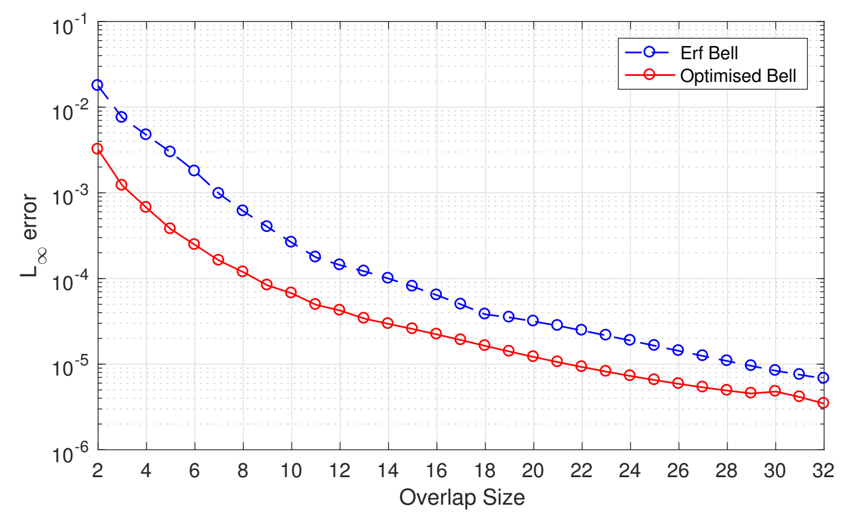
$$\frac{\partial p_i}{\partial t} = \mathscr{F}^{-1}\left\{ik_i \mathscr{F}\left\{b \cdot p_i\right\}\right\}$$

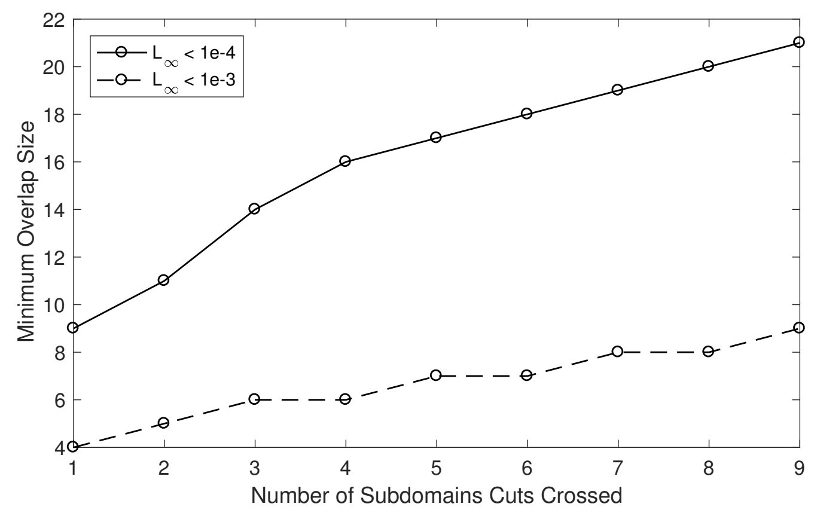




Local Fourier Basis Accuracy

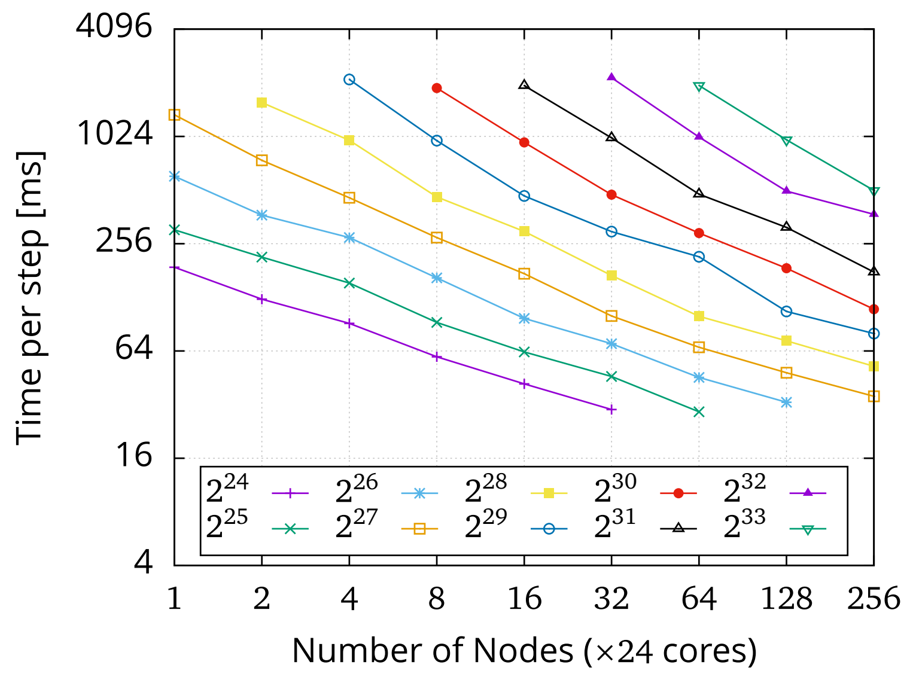
Since the gradient is not calculated on the whole data, numeric error is introduced. Its level can be tuned by the shape of the bell function and thickness of the overlap region. An overlap size of 8 grid points is then sufficient to maintain the L_∞ error below 0.1% even after the wave has crossed 8 subdomain boundaries. For 3D decompositions, this corresponds to $9^3 = 729$ local subdomains.

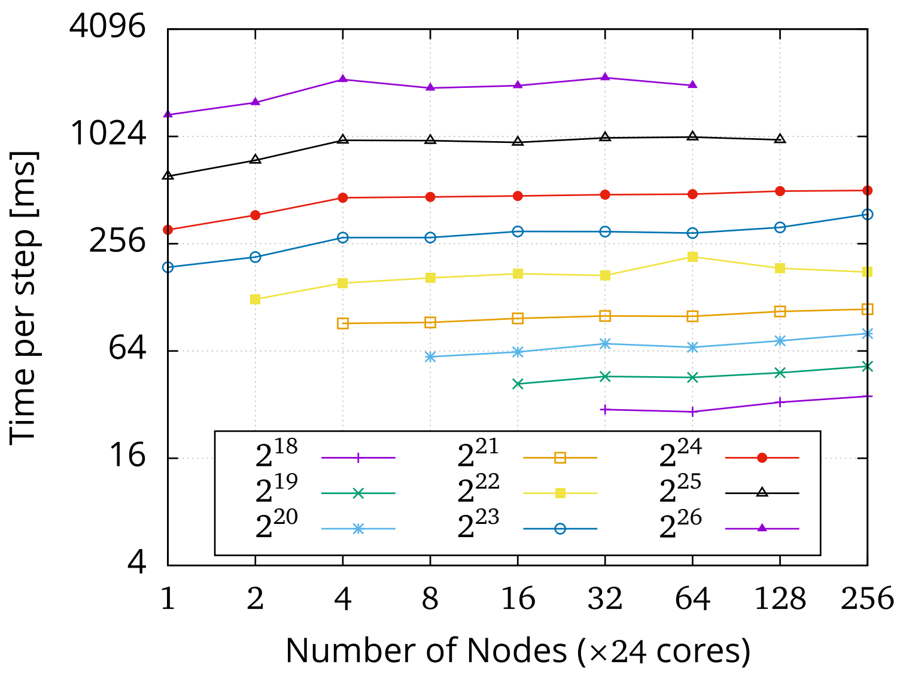




Performance Investigation

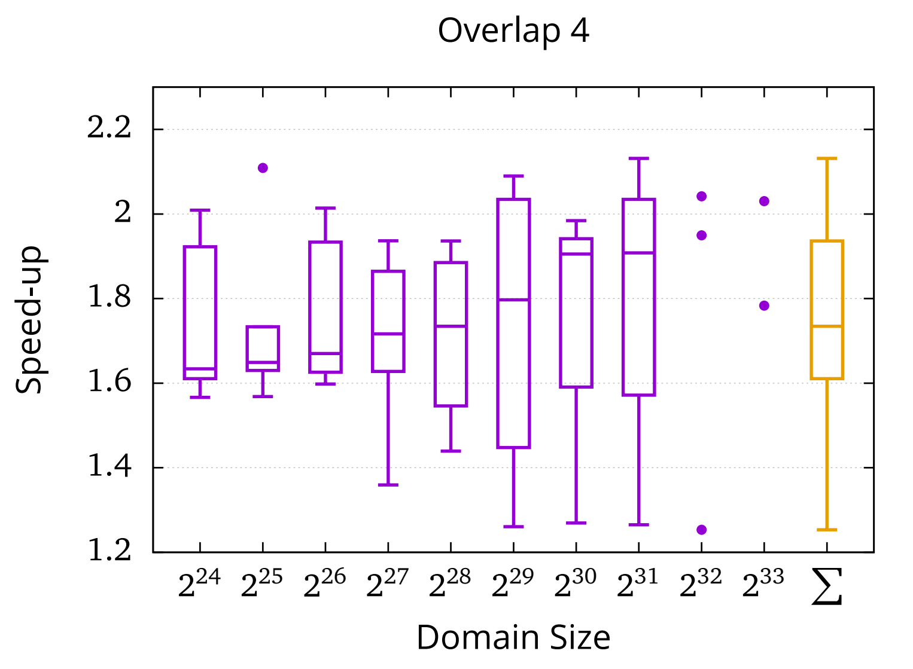
Strong and weak scaling of ultrasound simulations with local domain decomposition and overlap size of 16 on Salomon cluster using 256 nodes and Infiniband interconnect. Domain sizes between 256^3 and 2048^3 grid points.

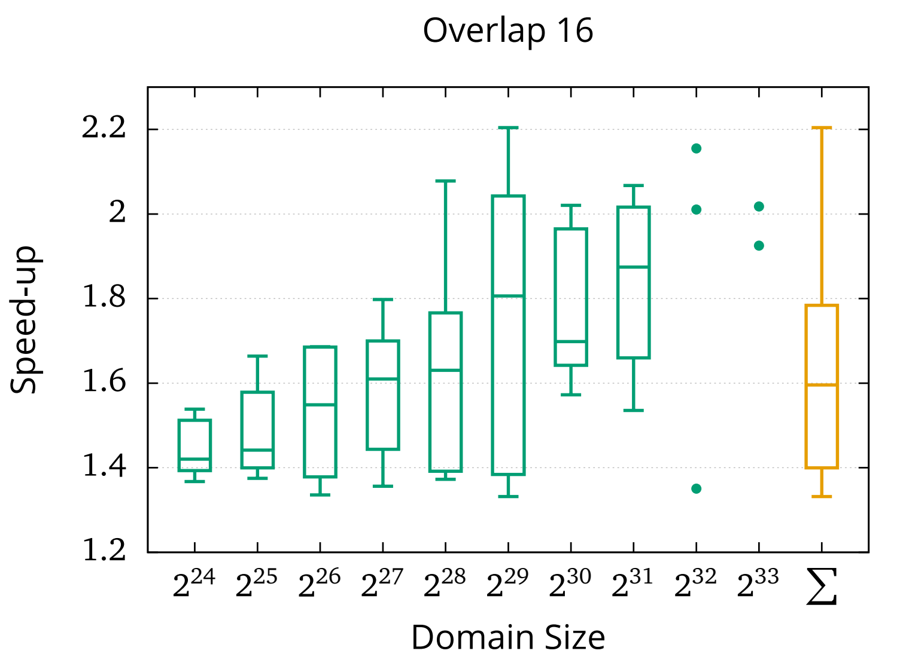




Overlap Size and Performance Trade-off

Gradient of scaling curves (speed-up when doubling the number of subdomains) for overlap sizes of 4 and 16 on Salomon cluster.





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